

# Global Warming Zonal Temp - Energy Balance Model

Energy flows into Earth through radiation from the Sun and out of Earth by reflection and radiation. There are latitudinal variations. The flow of energy into Earth and the flow of energy out of Earth must be equal if Earth is to maintain a stable temperature.

$H_0$  is the extraterrestrial solar flux ( $W/m^2$ ). In the zonal model, we need to be able to calculate the total energy received from the sun per unit time. This is given by  $\pi R^2 H_0$ . The average extraterrestrial solar flux over the entire surface can be calculated by  $H_0/4$ .

## Anthropogenic Global Warming Simulation:

The long wavelength flux to space,  $H_+(T, c)$  can be approximated by a first order expansion in the surface air temperature and the logarithm of the atmospheric carbon dioxide concentration,  $c$ .

$$H_+(T, c) = A c_{CO2} + B c_{CO2} T - C c_{CO2} \ln(c/c_0)$$

We will assume that  $A c_{CO2}$ ,  $B c_{CO2}$ , and  $C c_{CO2} \ln(c/c_0)$  are a component parts of our general model for this point in time. We will then explicitly separate out the  $C c_{CO2} \ln(c/c_0)$  contribution.

There are a number of estimates for  $C c_{CO2}$ . The preindustrial atmospheric CO2 concentration is 280 ppm. Today's value of 370 ppm

## Data Constants:

SC: Solar Constant ( $W/m^2$ ),

A & B: Long Wave Radiation Heat Loss (Greenhouse) A ( $W/m^2$ ), B ( $W/m^2/C$ )  $R_{loss} = A + B \times T$

C: Transport Coefficient (Conductivity between zones) C ( $W/m^2/C$ )

Tcrit: Temperature at which land becomes covered with snow and/or water turns to ice

Aice; Ice Albedo, SunWt: Annual mean radiation at latitude  $W/m^2$

CO2 Global Warming Forcing = 5.7  $W/m^2$

$$\begin{aligned} SC &:= 1370 & B &:= 2.17 & C &:= 3.87 & CO2 &:= 5.7 & A &:= 204 - CO2 & T_{crit} &:= -10 \\ SC_{frac} &:= 0.9 & \alpha_{ice} &:= 0.6 & \alpha_{land} &:= 0.295 & c_0 &:= 200 & c_x &:= 400 \\ max\_tilt &:= 23.5 & days\_in\_year &:= 365 & hours\_in\_day &:= 24 & zonal\_degrees &:= 360 \\ Pole\_Temp\_Diff\_Dat &:= -42.3 & Glob\_Avg\_Temp &:= 14.9 & C_{gw} &:= CO2 \cdot \ln\left(\frac{400}{c_0}\right) \end{aligned}$$

## Read NCEP surface air temperature (annual, zonal mean, deg C) and Sim Temp

TempDat := READPRN("NCEP\_air\_zonal.dat")      rows(TempDat) = 72

TempSimToday := READPRN("FinalTempSim.txt")

## Read Albedo Data: Earth Radiation Budget Experiment (ERBE) 1986 - 1989

AlbedoLat := READPRN("Avg\_Albedo\_Latitude.txt")      rows(AlbedoLat) = 72

lats := 36    lat := 1..lats    Init\_T\_lats := 100    LatA := AlbedoLat<sup>(1)</sup>    Albedo := AlbedoLat<sup>(2)</sup>

$zonal\_width := \frac{90}{lats}$        $ZoneLat_{lat} := \frac{zonal\_width}{2} + (lats - lat) \cdot zonal\_width$

$delta\_rad := \frac{\pi}{lats}$        $zones_{lat} := \frac{zonal\_width}{2} + (lat - 1) \cdot zonal\_width$

$lats\_rad := \frac{lats \cdot \pi}{180}$       max\_steps := 100

## Fraction of the surface of sphere in each latitude zone

$lats\_frac := (\sin(lats\_rad + delta\_rad) - \sin(lats\_rad - delta\_rad))$

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reverse(V) :=
┌ R ← rows(V)
│ for r ∈ 1..R
│   Rev_r ← V_{R+1-r}
└ Rev
```

**Daily rotation of earth reduces solar constant by distributing sun energy along a zonal band**

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total_solar := | total_solar ← 0
                | for hour ∈ 1..hours_in_day
                | | noon_angle ← zonal_degrees ·  $\frac{\text{hour}}{\text{hours\_in\_day}}$ 
                | | for longitude ∈ 1..zonal_degrees
                | | | sun_angle ← longitude - noon_angle
                | | | MaxAngle ← if  $\left(\cos\left(\frac{\pi \cdot \text{sun\_angle}}{180}\right) > 0, \cos\left(\frac{\pi \cdot \text{sun\_angle}}{180}\right), 0\right)$ 
                | | | total_solar ← total_solar + SC · MaxAngle
                | total_solar
solar_constant :=  $\frac{\text{total\_solar}}{\text{hours\_in\_day} \cdot \text{zonal\_degrees}}$ 
FlipAug(V) := | R ← rows(V)
               | for r ∈ 1..R
               | | VFr ← Vr
               | | VFR+r ← VR+1-r
               | VF

```

**Annual Insolation: Accumulate normalized insolation through a year**

Insolation<sub>lat</sub> := 0      Init\_T := FlipAug(Init\_T)

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insolation := | I ← Insolation
               | for day ∈ 1..days_in_year
               | | tilt ← max_tilt ·  $\cos\left(2 \cdot \frac{\pi \cdot \text{day}}{\text{days\_in\_year}}\right)$ 
               | | for j ∈ 1..lats
               | | | zenith ← if  $(\text{zones}_j + \text{tilt} < 90, \text{zones}_j + \text{tilt}, 90)$ 
               | | | Ij ← Ij +  $\cos\left(\text{zenith} \cdot \frac{\pi}{180}\right)$ 
               | | solar_constant · reverse(I)
               |  $\frac{\text{days\_in\_year}}$ 

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latsS := 2 · lats    latS := 1..latsS    insolation := FlipAug(insolation)    ZoneLat := FlipAug(ZoneLat)

SumCos :=  $\sum \cos\left(\text{ZoneLat} \cdot \frac{\pi}{180}\right)$       T\_Cos<sub>latS</sub> := Init\_T<sub>latS</sub> ·  $\cos\left(\text{ZoneLat}_{\text{latS}} \cdot \frac{\pi}{180}\right)$

Mean\_T :=  $\sum T_{\text{Cos}} \cdot \frac{1}{\text{SumCos}}$       Temp<sub>init</sub> :=  $\frac{[\text{insolation} \cdot (1 - \text{Albedo}) + C \cdot \text{Mean}_T - A - \text{Cgw}]}{B + C}$

Mean\_T = 4.362      mean(Temp<sub>init</sub>) = 5.462      Step<sub>latS</sub> := 0

tol\_temp\_diff := 1

```

Final_TA(Albedo) :=
  step ← 2
  max_temp_diff ← 100
  Temp ← Tempinit
  while step < max_steps ∧ max_temp_diff > tol_temp_diff
  |
  |   step ← step + 1
  |   Temp_old ← Temp
  |   Mean_Tstep ←  $\sum \left( \text{Temp} \cdot \cos \left( \text{ZoneLat} \cdot \frac{\pi}{180} \right) \right) \cdot \frac{1}{\text{SumCos}}$ 
  |   Temp ←  $\frac{\text{insolation} \cdot (1 - \text{Albedo}) + C \cdot \text{Mean\_T}_{\text{step}} - A - C_{\text{gw}}}{(B + C)}$ 
  |   for lat ∈ 1 .. latsS
  |   |
  |   |   Albedolat ← αice if Templat < Tcrit ∧ Albedolat < αland
  |   |   Albedolat ← αland if Templat > Tcrit ∧ Albedolat < αland
  |   |   Steplat ← step
  |   |
  |   |   max_temp_diff ← max  $\left[ \left| \frac{\text{Temp\_old} - \text{Temp}}{\text{Temp\_old} - \text{Temp}} \right| \right]$ 
  |   |
  |   |   Final_T ←  $\frac{\text{insolation} \cdot (1 - \text{Albedo}) + C \cdot \text{Mean\_T}_{\text{step}} - A - C_{\text{gw}}}{B + C}$ 
  |   |
  |   Final_T ← augment(Final_T, Albedo, Step)

```

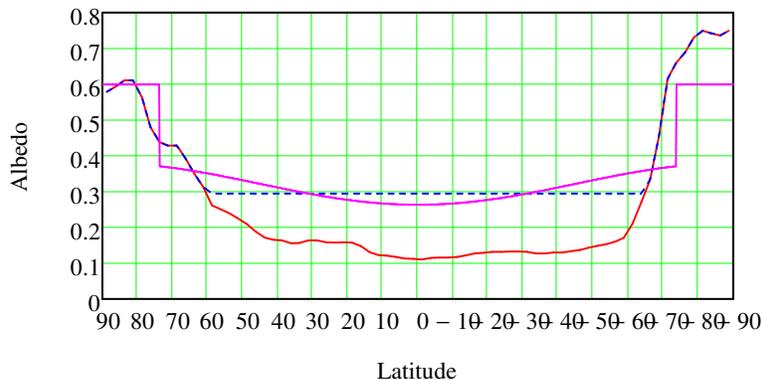
FTA := Final\_TA(Albedo)      FTA<sub>1,3</sub> = 12      Final\_Temp := FTA<sup><1></sup>  
 mean(Final\_Temp) = 5.134      mean(TempDat) = 3.089      ALBEDO := FTA<sup><2></sup>  
 NPole\_Eq\_Diff := Final\_Temp<sub>1</sub> - Final\_Temp<sub>lats</sub>      NPole\_Eq\_Diff = -44.562  
 Latitude **Cape Coral, FL**: 26° 33' 45" N. Avg Temp 23.5 C      Final\_Temp<sub>26</sub> = 18.478

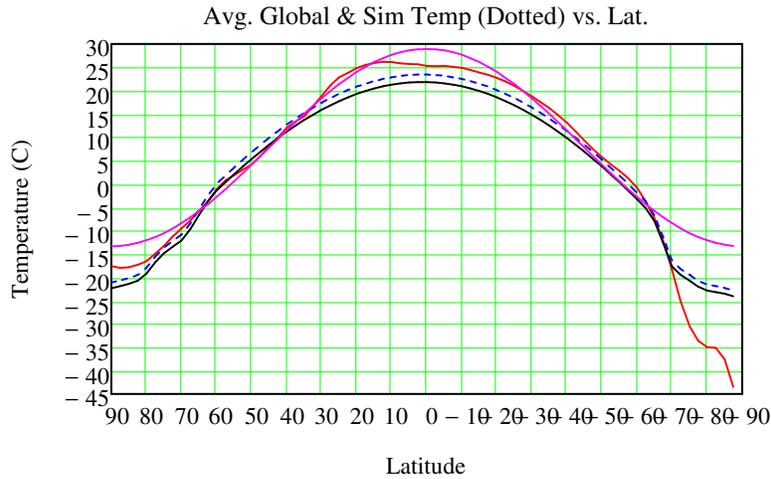
### **North's 1-D Effusive EBM Analytic Model (Magenta Plots)**

To is the planetary, globally averaged temperature, T<sub>2</sub> is 2/3 of the Temp difference from the poles to equator, T<sub>pe</sub>. The ice sheet edge (T = -10C) is above 73.74°, with ice albedo, α<sub>ice</sub>:

$$\begin{aligned}
 T_o &:= 14.9 & T_{pe} &:= -42.3 & T_2 &:= \frac{2}{3} \cdot T_{pe} & x_s &:= 0.96 \\
 \alpha_o &:= 0.303 & \alpha_2 &:= 0.0779 & \alpha(\theta) &:= \text{if} \left( |\theta| < 73.7, \alpha_o + \alpha_2 \cdot \text{Leg} \left( 2, \sin \left( \frac{\theta \cdot \pi}{180} \right) \right), \alpha_{ice} \right) \\
 T_{\text{North}}(\theta) &:= T_o + T_2 \cdot \text{Leg} \left( 2, \sin \left( \frac{\theta \cdot \pi}{180} \right) \right) & T_{\text{North}}(90) &= -13.3 & T_{\text{North}}(73.7) &= -9.968
 \end{aligned}$$

Albedo Data & Simulation Albedo (Dotted)





WRITEPRN("FinalTempSim.txt") := Final\_Temp

### Climate Sensitivity: Anthropropic Global Warming from Doubled CO2 Concentration

AGW := mean(Final\_Temp - TempSimToday)      AGW = 1.444

Taking the central value of the likely range of climate sensitivities of 3 oC (the temperature rise from doubling atmospheric CO2) the equilibrium temperature rise is expected to be:

$$\text{Temp rise} = \ln\left(\frac{[\text{CO}_2]_2}{[\text{CO}_2]_1}\right) \cdot \frac{3}{\ln 2}$$

where [CO2]1 is the starting [CO2] level, [CO2]2 is the end [CO2] level, 3 is the climate sensitivity and ln2 refers to the doubling.

$$\Delta T_{\text{CO}_2}(\text{CO}_2) := \ln\left(\frac{\text{CO}_2}{c_0}\right) \cdot \frac{3}{\ln(2)}$$

H2O and CO2 have the same number of IR active vibrations - 3. An absorption band at a given frequency is due to a specific vibration. For example the 650cm<sup>-1</sup> CO2 absorption is due to the (doubly degenerate) bending mode. So more asymmetry in a molecule does not mean more absorption at a given wavelength. It usually means more wavelengths at which it absorbs.

However asymmetry will complicate the rotational-fine structure of a particular IR absorption, however H2O's rotational fine structure will be quite dispersed because of its light mass (18 compared to CO2's 44)

### Radiative Forcing

The radiative forcing for CO2 (this is the forcing at the tropopause, not the surface, as described in Myhre et al 1998 and later papers) is

$$k := 5.35 \cdot \frac{\text{W}}{\text{m}^2} \quad F_{\text{CO}_2}(\text{CO}_2) := k \ln\left(\frac{\text{CO}_2}{c_0}\right) \quad F_{\text{CO}_2}(2 \cdot c_0) = 3.708 \frac{\text{kg}}{\text{s}^3}$$

Where the constant k (derived from line-by-line radiative transfer codes) C0 are the final and initial CO2 concentrations.

Climate sensitivity is the temperature response of the system per unit forcing. In other words, a high climate sensitivity means that it is very easy to change the global mean temperature, while a very low sensitivity would require an enormous forcing to get that same change. In the easiest case, we'll consider what happens when you only increase some forcing (say double CO2) and allow the outgoing radiation to increase (according to the Stefan-Boltzmann law) to re-establish a new radiative equilibrium. Here, nothing else changes with the climate state (no cloud cover changes, no ice melts, etc) except for our forcing. This is the so-called Planck response.