Empirical Model: ENSO, Solar, Volcanic Aero, Anthro

http://www.leapcad.com/Climate_Analysis/Empirical_Model_ENSO_Solar_VolcAero_Anthro.xmcd

On time scales of 10 to 50 years (and longer) decadal climate forecasts are difficult to make with general circulation climate models due to their many uncertainties [IPCC, 2007]. We will use the methodology first used by Schonwiese and Bayer in "Some statistical aspects of anthropogenic and natural forced global temperature change", 1995. We will model global temperature by with a multiforced lagged regression by combining ENSO, Volcanic Aerosols, Irradiance, and the effects of Anthropogenic Influence.

Analysis

Using the most recently available characterizations of ENSO, E, volcanic aerosols, V, solar irradiance, S, and anthropogenic influences, A, we perform multiple linear regression analysis to decompose monthly mean surface temperature anomalies since 1980 into four components.
Monthly mean surface temperature anomalies ΔT_{MS} are reconstructed as:

$$\Delta T_{MS}(t) = co + c_E \cdot E\left(t - \Delta t_E\right) + c_V \cdot V\left(t - \Delta t_V\right) + c_S \cdot S\left(t - t_S\right) + c_A \cdot A\left(t - t_A\right)$$

Where E, V, S and A are the time series and the lags (in months) are $\Delta t_E = 3$, $\Delta t_V = 6$, and $\Delta t_S = 0$ and $\Delta t_A = 17$ years. The lags are chosen to maximize the proportion of global variability that the statistical model captures and are spatially invariant (although a geographical dependence is expected). The fitted coefficients, c_o. . ., are obtained by multiple linear regression against the instrumental surface temperature record (HadCRUT3v).

The multivariate ENSO index, E, is a weighted average of the main ENSO features contained in sea-level pressure, surface wind, surface sea and air temperature, and cloudiness [Wolter and Timlin, 1988]. Volcanic aerosols, V, in the stratosphere are compiled by Sato et al. [1993] since 1850, updated from giss.nasa.gov to 1999 and extended to the present with zero values. Although some volcanic activity occurred between 2006 and 2008, it is difficult to calculate the aerosol optical depth because of the lack of direct quantitative space-based observations. Solar irradiance, S, is estimated as the competing effects of sunspots and facular, identified in observations made by space-based radiometers [Lean et al., 2005]. The anthropogenic influence, A, is the Forcing Effect of the concentration (ppm) of CO_2.

Climate Forecasting:

Using global and regional surface temperature responses to the four individual influences parameterized by regression against the observations from 1980 to 2008, we forecast change from 2009 to 2020 by adopting the best estimate of how each influence will change in the future. The anthropogenic forcing in the past 40 years is well represented by a linear trend that we extrapolate into the future.

We assume that future solar irradiance cycles replicate cycle 23, with cycle 24 (See: **http://www.leapcad.com/Climate_Analysis/Climate_Data-Proxies_and_Reconstructions.pdf** pg. 21) commencing at the beginning of 2009. Although solar activity (as indicted by sunspot numbers) was less in cycle 23 than in cycles 21 and 22, the total irradiance amplitude (near 0.1%) is similar in the three past cycles since it is the net effect of sunspot darkening and facular brightening, both of which are altered by solar activity. Since ENSO fluctuations and volcanic eruptions are not predictable on decadal time scales, we estimate their maximum likely future impact with a scenario that includes a Pinatubo-like eruption with peak impact in 2014 and a super ENSO with maximum impact in 2019, mimicking a similar sequence that occurred from 1992 to 1997 (Figure Stratospheric Optical Depths).

The Data: Temperature, ENSO Index, Volcanic Aerosols, Anthropogenic (CO2 ppm)

HadCrut Temperature and CO2 ppm

http://www.cru.uea.ac.uk/cru/data/temperature/hadcrut3vgl.txt Monthly Temp Data 1850 to 2009

Read data from http://www.esrl.noaa.gov/gmd/ccgg/trends/co2_data_mlo.

MLCO2 := READPRN("NOA _Mauna_Loa_Monthly_CO2.TXT")

$Date := MLCO2^{\langle 2\rangle}$ $\quad CO2_{ML} := MLCO2^{\langle 4\rangle}$ $\quad TrendCO2 := MLCO2^{\langle 5\rangle}$ $\quad RD := rows(Date)$

Get CO2 Trend Line from 1990 to 2010, then project to 2020

$Date_{1990} := submatrix(Date, 383, RD-1, 0, 0)$ $\quad L_{co2} := line\left(Date_{1990}, submatrix(TrendCO2, 383, RD-1, 0, 0)\right)$

$Trend_{co2}(Year) := L_{co2_0} + L_{co2_1} \cdot Year$ $\quad Co := 280$ $\quad m := 0..12 \cdot 10$ $\quad Yr2020_m := 2010 + \frac{m}{12}$

$$Keeling(yr) := 1.054 \cdot 10^{-2} (yr - 1960)^2 + 9 \cdot 10^{-1} \cdot (yr - 1960) + 315.5$$

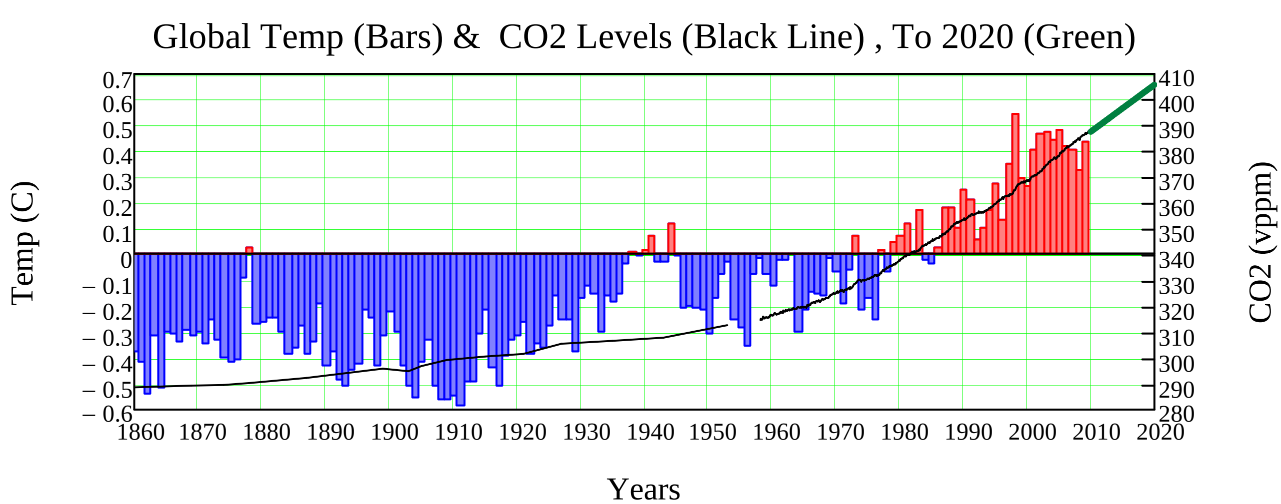
SipleCO2 := READPRN("Friedli Siple CO2 1986.TXT") IceCO2 := READPRN("CO2 Ice Core Data.txt")

http://www.wasserplanet.becsoft.de/180CO2/CO2tot1812-2007.txt
Column C: CO2 total 1812-1961 corrected. annual averages from raw data.

HadCrut := READPRN("hadcrut3vgl.txt") rows(HadCrut) = 320 cols(HadCrut) = 14 n := 0..159
HadCrutx := READPRN("hadcrut3glx.txt") rows(HadCrutx) = 160 cols(HadCrutx) = 14 n := 0..159

$$TCrut_n := \sum_{m=1}^{12} \left(HadCrut_{2\cdot n, m} \cdot \frac{1}{12} \right) \qquad Time_{crut_n} := HadCrut_{2\cdot n, 0}$$

$$TCrut_n := HadCrutx_{n,13} \qquad TCrutPlus := \overrightarrow{(\Phi(TCrut) \cdot TCrut)}$$



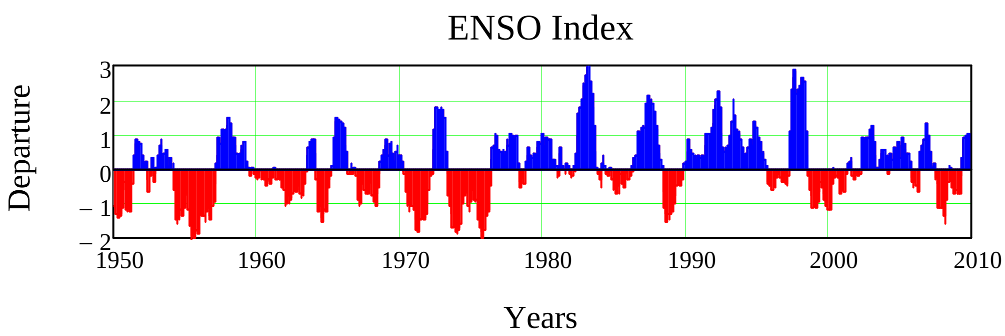
ENSO Index Monthly Data from 1950 to 2009

http://www.esrl.noaa.gov/psd/people/klaus.wolter/MEI/mei.html Monthly ENSO Data 1850 to 2009

MEIM := READPRN("MultiVariate ENSO Index.TXT")   R := rows(MEIM)   rr := 0 .. (R − 1)·12 + 11

MEIx := submatrix(MEIM, 0, R − 1, 1, 12)

$$MEI_{rr} := MEIx_{floor\left(\frac{rr}{12}\right), mod(rr,12)} \qquad MEID_{rr} := \left(MEIM^{\langle 0 \rangle}\right)_{floor\left(\frac{rr}{12}\right)} + \frac{mod(rr,12)}{12}$$



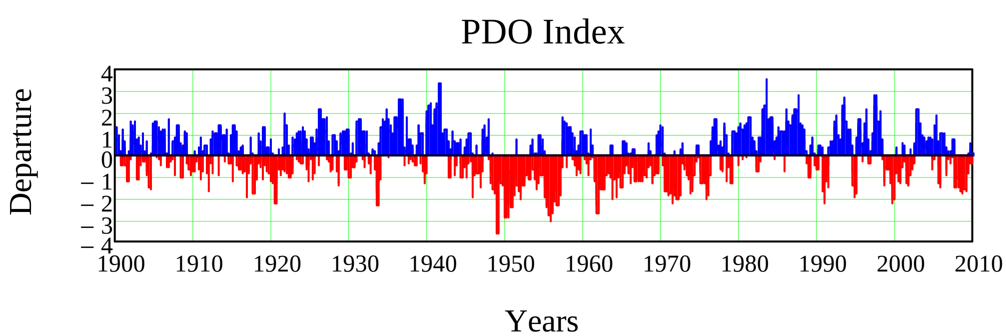
Pacific Decadal Oscillation (PDOI) Index from 1900 to 2009

Note: ENSO and PDO are not statistically independent. They have a 47% correlation.

http://jisao.washington.edu/data/pdo/   Year, Jan to Dec

PDO := READPRN("PDO_Index.dat")   R := rows(PDO)   rr := 0 .. (R − 1)·12 + 11

$$PDOI_{rr} := PDOx_{floor\left(\frac{rr}{12}\right), mod(rr,12)} \qquad PDOYr_{rr} := \left(PDO^{\langle 0 \rangle}\right)_{floor\left(\frac{rr}{12}\right)} + \frac{mod(rr,12)}{12}$$

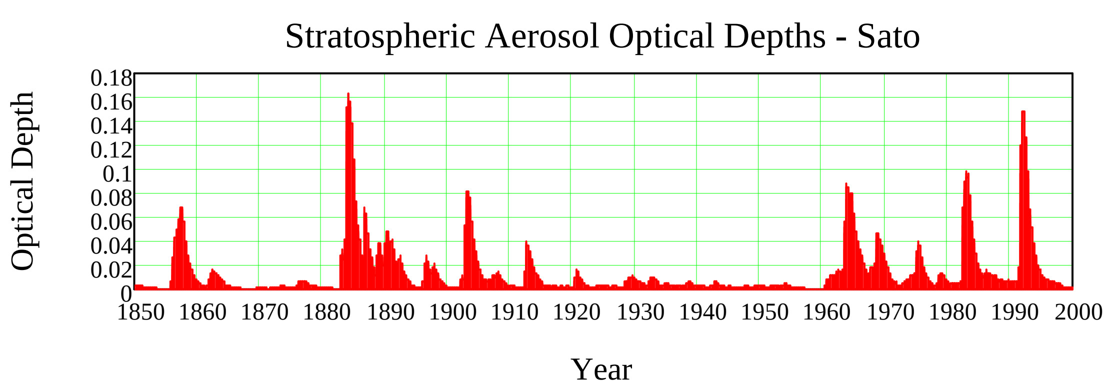


Stratospheric Volcanic Aerosols

Note: Volcanic Aerosols and ENSO are not statistically independent. Corr = 40%

http://data.giss.nasa.gov/modelforce/strataer/   Data: Global, NH, SH

VA := READPRN("Aerosols-Monthly-Mean Optical Thickness_tau_line.dat")



PMOD Solar Irradiance

$TSI_{PMOD} := READPRN(\text{"TSI from 1979 to 2009-PMOD composite\_d41\_62\_0906.txt"})$

$TSI := READPRN(\text{"TSIpmod2.txt"})$

$DateTSI := READPRN(\text{"TSIpmodDate.txt"})$

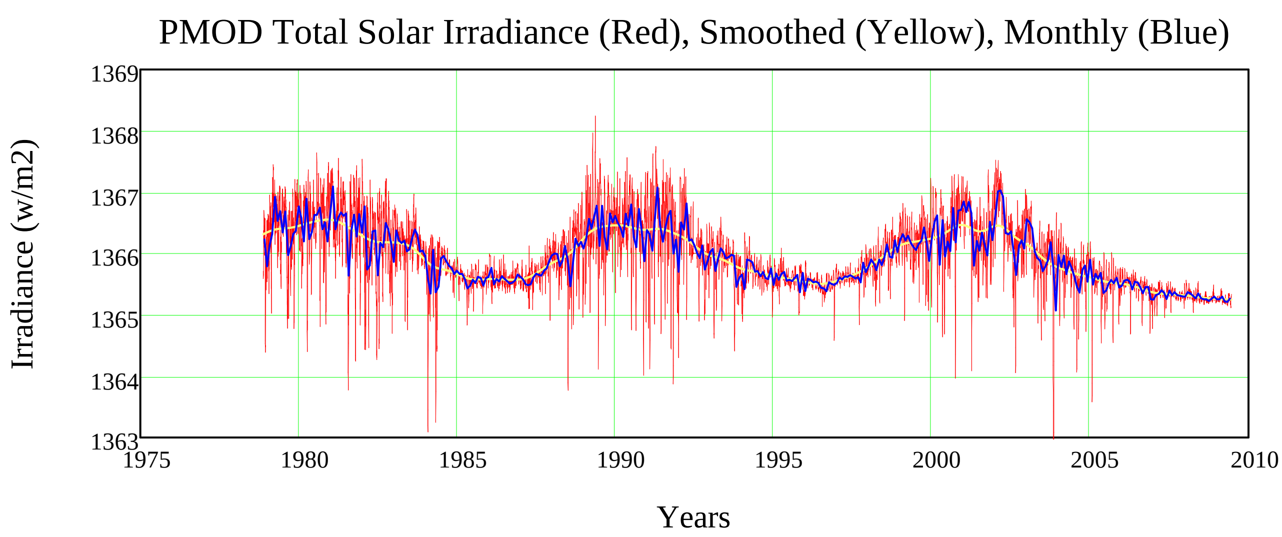
$TSIYr := 1980 + floor\left(TSI_{PMOD}^{\langle 1 \rangle} \cdot 365^{-1}\right)$

$TSIMon := ceil[(DateTSI - TSIYr) \cdot 12]$

TAM, Convert Daily to Monthly Average

$TSIMonAvg := TAM(TSIMon, TSIYr, TSI)$

$TSI_{smm} := READPRN(\text{"TSIsm8.txt"})$



Reconstruction of Solar Irradiance since 1610, Lean 1995 (1600-1995

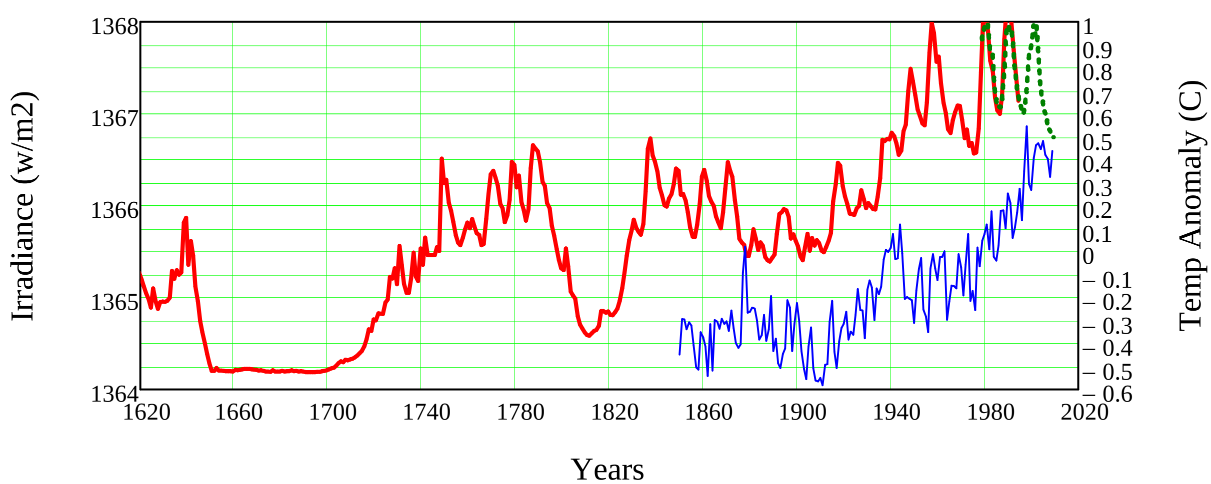
ftp://ftp.ncdc.noaa.gov/pub/data/paleo/contributions_by_author/lean1995/

$TSD_{lean} := READPRN(\text{"TSDLeanFilled.txt"})$

$TSDF_{lean} := READPRN(\text{"lean1995data.txt"})$ $\quad Yr_{lean} := TSDF_{lean}^{\langle 0 \rangle}$

Solar Irradiance Correlates with U.S. Temp Anomaly

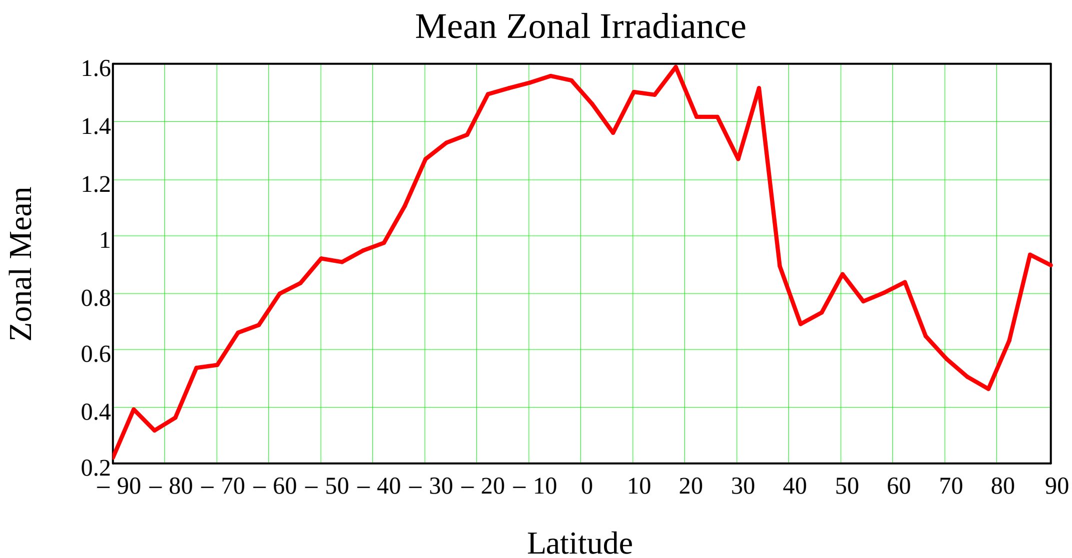
Lean Reconstructed Solar Irradiance (Red), PMOD +1.5 (Green), and Temp Anomaly (Blue)



Zonal Variation of Irradiance

http://data.giss.nasa.gov/cgi-bin/cdrar/effij.py

IrradZonal := READPRN("Irradiance zonal01.dat")



Monthly Time Series Matrices with Optimized Lags for Period 1980 - 2005

Let Y be the Temp and X1, X2, X3, and X4 be the delayed matrices for ENSO, Irradiance, Volcanic Aerosols, and Anthropogenic Influence.

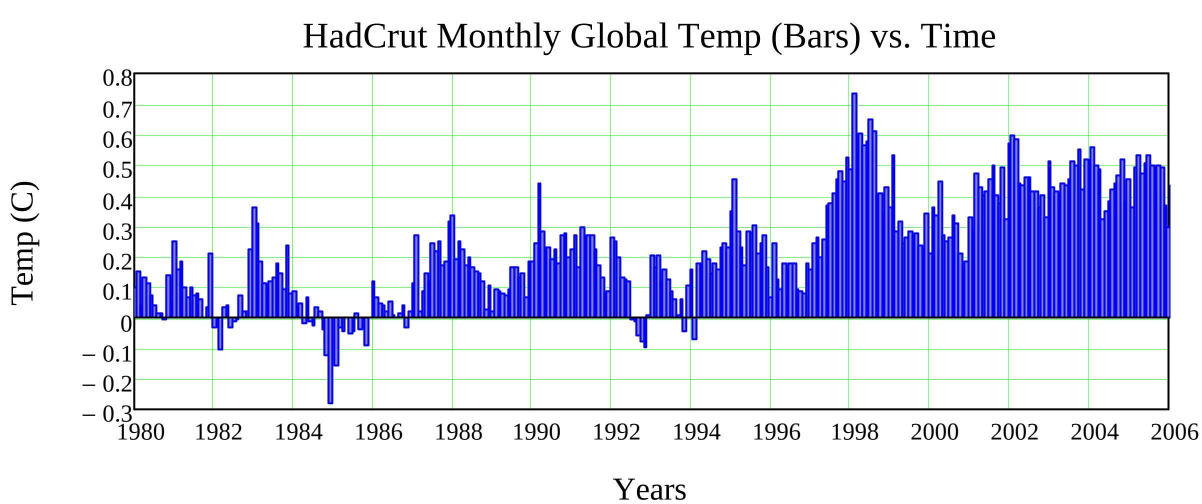
HadCrut Temperature Data

$HadCrut_{260,0} = 1980$ $\quad 2005 - 1980 = 25$ $\quad r25 := 0..26$ $\quad YrEven_{r25} := 260 + 2 \cdot r25$ $\quad n := 0..12$

$HadCrutDat_{r25,n} := HadCrut_{260+r25 \cdot 2,n}$ $\quad$ HadCrutMDat $\quad R := rows(HadCrutDat)$ $\quad rr := 0..(R-1) \cdot 12 + 11$

$$HCTime_{rr} := \left(HadCrutDat^{\langle 0 \rangle}\right)_{floor\left(\frac{rr}{12}\right)} + \frac{mod(rr,12)}{12} \qquad HCTemp_{rr} := HadCrutDat_{floor\left(\frac{rr}{12}\right), mod(rr,12)+1}$$

$Y := submatrix(HCTemp, 0, 12 \cdot 26, 0, 0)$ $\quad Year_Y := submatrix(MEID, 360, 360 + 12 \cdot 26, 0, 0)$ $\quad MEID_{360} = 1980$



Empirical Component Data for 1980 to 2005, X1, X2, X3, X4, Given Optimized Lags (Months), Δt:

(The Optimization Procedure and Optimized Month Delays (3, 6, 0, 17Yrs) follows on pg 7.)

(follows on pg 7.)

Lag Operator Δt(X, N=1980, τ): $\Delta t(X, N_{1980}, \tau) := \text{submatrix}(X, N_{1980} - \tau, N_{1980} - \tau + 12\cdot 26, 0, 0)$

Define Forcing Parameters: X are normal forcings, Xs are Gasussian Smoothed

X1 = ENSO (E): 1980 through 2005 Δt_E = 3, $MEID_{360} = 1980$ $\quad x1 := \Delta t(MEI, 360, 3)$

$\mu 1 := \text{mean}(x1) \quad \sigma 1 := \text{stderr}(Year_Y, x1) \quad z1 := (x1 - \mu 1)\cdot \sigma 1^{-1} \quad xs1 := \text{ksmooth}(Year_Y, z1, 2)$

X2 = Monthly Volcanic Aerosols (V), τ_V = 6, $VA_{1560,0} = 1980.04$

$m := 0..100 \quad Zeros_m := 0 \quad VAZ := \text{stack}(VA^{\langle 1\rangle}, Zeros) \quad x2 := \Delta t(VAZ, 1560, 6)$

$\mu 2 := \text{mean}(x2) \quad \sigma 2 := \text{stderr}(Year_Y, x2) \quad z2 := (x2 - \mu 2)\cdot \sigma 2^{-1} \quad xs2 := \text{ksmooth}(Year_Y, z2, 2)$

X3 = Solar Irradiance (S), τ_S = 0 $\left(TSIMonAvg^{\langle 0\rangle}\right)_{13} = 1980$

$x3 := \Delta t(TSIMonAvg^{\langle 1\rangle}, 13, 0) \quad \mu 3 := \text{mean}(x3) \quad \sigma 3 := \text{stderr}(Year_Y, x3) \quad z3 := (x3 - \mu 3)\cdot \sigma 3^{-1}$

$xs3 := \text{ksmooth}(Year_Y, z3, 2)$

X4 = Anthropogenic Forcing, ΔF, of CO2 ppm (A), τ_A = 10*17 (17 Yr Delay)

Assume total radiative forcing (includes CH4) is proportional to radiative forcing due to carbon dioxide.

$\Delta F(C) := 4.841 \ln\left(\frac{C}{Co}\right) + 0.0906\cdot\left(\sqrt{C} - \sqrt{Co}\right) \quad \Delta F_{IPCC}(C) := 6.3 \ln\left(\frac{C}{Co}\right)$

$CO2_{ppm} := \Delta t(TrendCO2, 261, 17\cdot 12) \quad x4 := \overrightarrow{\Delta F_{IPCC}(CO2_{ppm})} \quad Date_{262} = 1980.042$

$\mu 4 := \text{mean}(x4) \quad \sigma 4 := \text{stderr}(Year_Y, x4) \quad z4 := (x4 - \mu 4)\cdot \sigma 4^{-1} \quad xs4 := \text{ksmooth}(Year_Y, z4, 2)$

X5 = Pacific Decadal Oscillation (PDO) Index (PDO), τ_D = 10*17) $PDOYr_{960} = 1980$

$x5 := \Delta t(PDOI, 960, 3) \quad \mu 5 := \text{mean}(x5) \quad \sigma 5 := \text{stderr}(Year_Y, x5) \quad z5 := (x5 - \mu 5)\cdot \sigma 5^{-1}$

Multi-Variate Component Construction and Design Matrix (Optimized Lags):

$$Y_i = \beta_0 + \beta_1\cdot X1_i + \beta_2\cdot X2_i + \beta_3\cdot X3_i + \beta_4\cdot X4_i + \varepsilon_i$$

The design matrix for our Temp Stats data can be constructed with the components $\quad i := 0..313 - 1$

$ONE_i := 1$	**ENSO**	**Volcanic Aerosols**	**Solar**	**Anthro - Effects CO2**
$X^{\langle 0\rangle} := ONE$	$X^{\langle 1\rangle} := z1$	$X^{\langle 2\rangle} := z2$	$X^{\langle 3\rangle} := z3$	$X^{\langle 4\rangle} := z4$
$Xs^{\langle 0\rangle} := ONE$	$Xs^{\langle 1\rangle} := xs1$	$Xs^{\langle 2\rangle} := xs2$	$Xs^{\langle 3\rangle} := xs3$	$Xs^{\langle 4\rangle} := xs4$

$b := \left(X^T\cdot X\right)^{-1}\cdot\left(X^T\cdot Y\right) \quad b^T = (0.21959 \quad 0.06608 \quad -0.05037 \quad 0.03063 \quad 0.01255)$

$bs := \left(Xs^T\cdot Xs\right)^{-1}\cdot\left(Xs^T\cdot Y\right) \quad bs^T = (0.21978 \quad 0.14923 \quad -0.09635 \quad 0.04577 \quad 0.01226)$

Save Model β Coefficient Results: WRITEPRN("Emp_ESVA_Coefficients.txt") := b

$\Delta T := b_0 + b_1\cdot X^{\langle 1\rangle} + b_2\cdot X^{\langle 2\rangle} + b_3\cdot X^{\langle 3\rangle} + b_4\cdot X^{\langle 4\rangle} \quad \Delta Ts := bs_0 + bs_1\cdot xs1 + bs_2\cdot xs2 + bs_3\cdot xs3 + bs_4\cdot xs4$

$\text{corr}(Y, \Delta T) = 0.87445 \quad RSquare := \text{corr}(Y, \Delta T)^2 = 0.76467 \quad \text{corr}(Y, \Delta Ts)^2 = 0.77449$

Optimization Procedure and Results:

Determine Time Lags to Maximize Correlation (R^2) of Regression Model to Global Temperature

Evaluate lags of 0 to 12 Months for ENSO, Aerosols, and Irradiance and 5 to 20 yrs for Effects of CO2 ppm

```
OptLags(Y) := | X<0> ← ONE
              | col ← MaxCorr ← 0
              | for te ∈ 3..3
              |   for ta ∈ 6..6
              |     for ti ∈ 0..2
              |       for tc ∈ 15..20
              |         | X<1> ← Δt(MEI, 360, te)
              |         | X<2> ← Δt(VAZ, 1560, ta)
              |         | X<3> ← Δt(TSIMonAvg<1>, 13, ti)
              |         | co2ppm ← Δt(TrendCO2, 261, tc·12)
              |         | X<4> ← vectorize(ΔF_IPCC(co2ppm))
              |         | b ← (X^T·X)^(-1)·(X^T·Y)
              |         | ΔT ← b_0 + b_1·X<1> + b_2·X<2> + b_3·X<3> + b_4·X<4>
              |         | Corr ← corr(Y, ΔT)^2
              |         | if Corr > MaxCorr
              |         |   | Opt_1,0 ← col
              |         |   | MaxCorr ← Corr
              |         | Opt<col> ← (Corr  te  ta  ti  tc)^T
              |         | col ← col + 1
              | Opt
```

Run Opt Routine and Gather Data

$OptDat := OptLags(Y) \quad OptDat_{1,0} = 2$

$OptDatT := OptDat^{T}$

$\left(OptDat^{\langle OptDat_{1,0}\rangle}\right)^{T} = (0.76467 \quad 3 \quad 6 \quad 0 \quad 17)$

$\max\left(OptDatT^{\langle 0\rangle}\right) = 0.76467$

Optimization Results

For Optimum E, V, S, A Lags:
3, 6, 0, 17 (yrs), respectively.
$R^2 = 0.76$

<u>The regression and estimation results are:</u>

The numbers shown in parentheses below the regression coefficients are the magnitudes of their t-ratios; i.e. the coefficients divided by the standard deviation of the regression coefficient. All but the coefficient for Sunspot number are significantly different from zero at the 95 percent level of confidence and they are of the right sign.

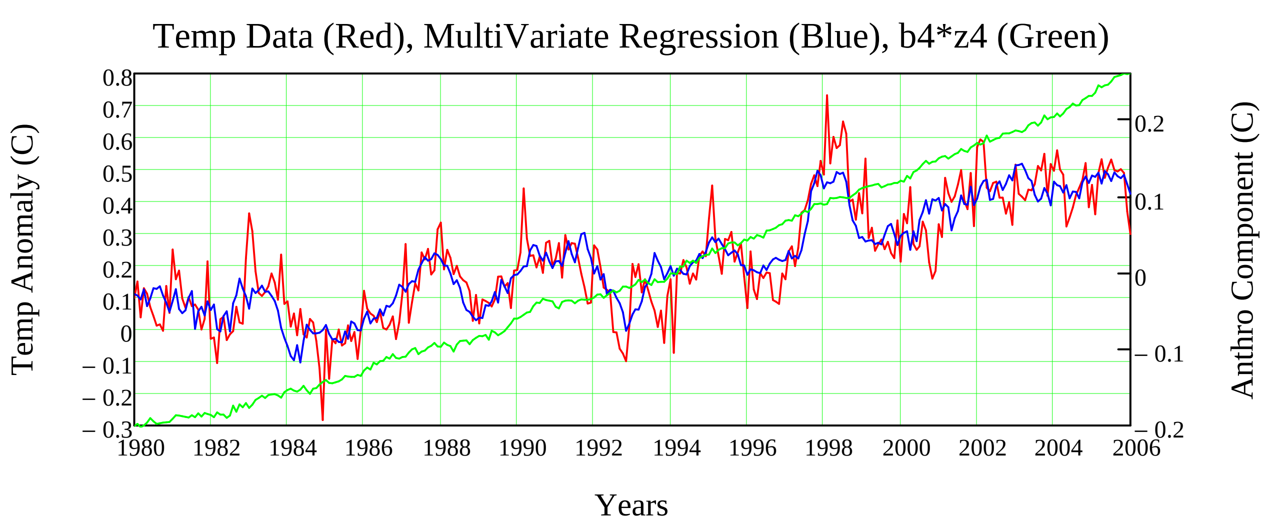
Shown below is a comparison of the observed temperature change and the temperature change predicted by the regression equation. The observations are shown in red and the estimations from the regression equation are shown in blue.

Another way of viewing the comparison is in the scatter diagram below of the actual and regression predicted temperature changes.

The t-ratios for the variables included in the regression equation are significant. They **explain <u>76 percent</u> of the variation in the year-to-year temperature change**. The insolation and CO_2 ppm both a 76% correlation. Also the effect of the CO_2 in the equation includes the effects of all variables influencing temperature change which are correlated with the general trend on CO_2 concentration but are not in the equation. These would i**nclude the effects of anthropogenic water vapor and anthropogenic cloudiness.**

<u>Compare Anthropogenic Forcing Component - (17 Year Lag) = b4*z4 (Green) of ΔT to Data</u>

<u>b4*z4 is the Optimized Match of Effects of Antro Forcing to Global Temperature Data</u>

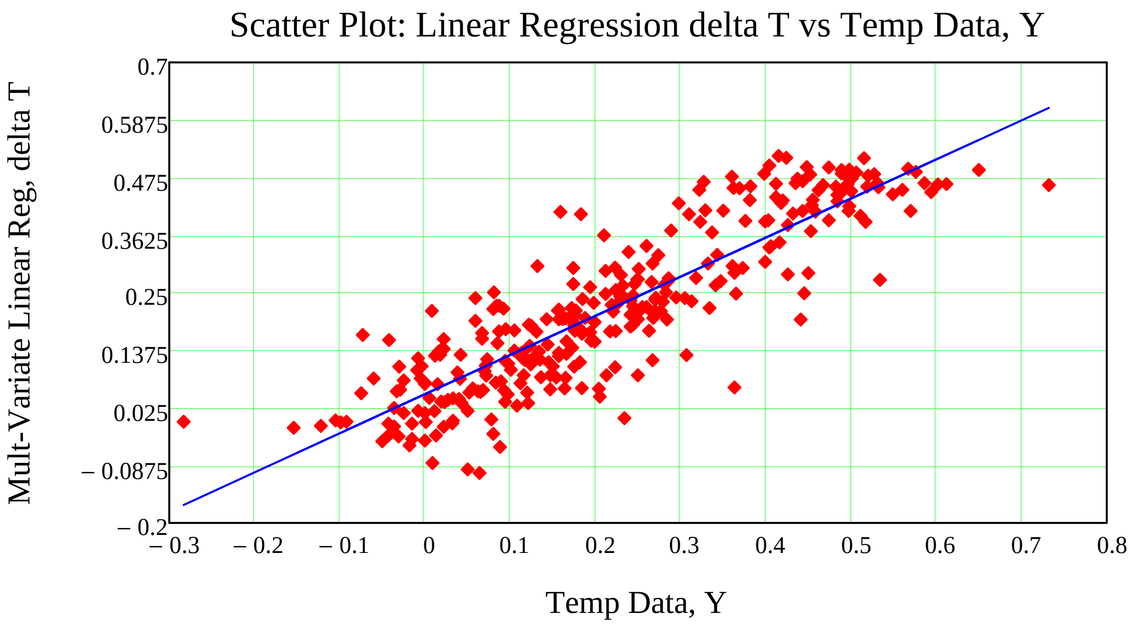


Statistical Analysis:

Four Factors - ENSO, Volcanic Aerosols, Insolation, and CO2 ppm explain 3/4 (76%) of the temperature variation.

$\text{int} := \text{intercept}(Y, \Delta T)$ $\qquad$ $s := \text{RSquare}$

$\text{int} = 0.05168$ $\qquad$ $s = 0.76467$ $\qquad$ $\delta T := \text{int} + s \cdot Y$



Test for Possible Regression

By extending this test to include p slope parameters $\qquad$ $\mathbf{H_0}$: $\beta_1 = \beta_2 = \beta_3 = \ldots = \beta_p = 0$

we have the equivalent test for the possibility of a multiple regression, $\qquad$ $\mathbf{H_0}$: no multiple regression relationship

As in simple linear regression, we can associate each data value with three types of deviations, specifically, the residual error, e

$\text{yhat} := X \cdot b$ $\qquad$ $e := Y - \text{yhat}$

Sum of Squares

We can also use matrices to calculate the sum of squares for residual error,

$SSE := e^T \cdot e$ $\qquad$ $SSE = 2.36546$

as well as for regression, $\qquad$ $SSR := \text{yhat} - \text{mean}(Y)^T \cdot (\text{yhat} - \text{mean}(Y))$

The total sum of squares equals $\qquad$ $n := \text{rows}(Y)$ $\qquad$ $p := 4$

$SST := SSE + SSR$ $\qquad$ $DF\_REG := p$ $\qquad$ $DF\_RESID := n - (p + 1)$ $\qquad$ $DF\_RESID = 308$

Mean Squares

Again, as in simple linear regression, dividing each sum of squares by the corresponding degrees of freedom provides us with variance estimates. The mean square for residual error

$$MSE := \frac{SSE}{DF\_RESID} \qquad MSR := \frac{SSR}{DF\_REG} \qquad DF\_TOTAL := n - 1$$

F Test

The final entry in the table is the F statistic and corresponding p-value for the significance of an overall multiple regression. Under the null hypothesis of

$\mathbf{H_0}$: no regression relationship

the test statistic $F := \frac{MSR}{MSE}$ $\qquad Rsq := \frac{SSR}{SST}$ $\qquad Rsq = 0.76467$

has an F distribution with $n1 := DF\_REG$ $\qquad n2 := DF\_RESID$

degrees of freedom. The p-value of the test, then, is given by

$$p\_val := 1 - pF(F, n1, n2) \qquad p\_val = 0$$

Summary: Analysis of Variance Table

Summarizing the above for our example,

DF	SS	MS	F
DF_REG = 4	SSR = 7.68624	MSR = 1.92156	F = 250.20084
DF_RESID = 308	SSE = 2.36546	MSE = 0.00768	p-value
DF_TOTAL = 312	SST = 10.0517		p_val = 0

The amount of variability explained by the linear regression (MSR) is greater than the amount due to residual error (MSE). The difference is large enough (the p-value is, in fact, close to 0) to strongly reject the null hypothesis,

Correlations between each pair of variables

in the model can be displayed in matrix form as

$$FLEX := augment\left(X^{\langle 1\rangle}, X^{\langle 2\rangle}, X^{\langle 3\rangle}, X^{\langle 4\rangle}, Y\right)$$

$$j := 0..p \qquad k := 0..p$$

$$CORR_{j,k} := corr\left(FLEX^{\langle j\rangle}, FLEX^{\langle k\rangle}\right)$$

STATISTICAL CORRELATIONS TO COMPONENTS

Correlation between "independent" variables

41% correlation between x1 (ENSO) and x2 (Volcanic)

Strong Correlation to Global Temp between:

Y (Temp) and x2 (Volcanic Aero) = 38%

Y (Temp) and x4 (Anthropogenic) = 78%

CORR =

	x1	x2	x3	x4	y
x1	1	0.41176	−0.0859	−0.10573	0.1549
x2	0.41176	1	−0.03022	−0.31352	−0.3816
x3	−0.0859	−0.03022	1	−0.15104	0.03587
x4	−0.10573	−0.31352	−0.15104	1	0.78122
y	0.1549	−0.38169	0.03587	0.78122	1

Evaluate t tests

$Var\_Covar\_b := \left(X^T \cdot X\right)^{-1} \cdot MSE$ $\quad$ $k := 0 .. p$

$se\_b_k := \overrightarrow{\sqrt{Var\_Covar\_b_{k,k}}}$ $\quad$ $se\_b^T = (0.00495 \quad 0.00544 \quad 0.00549 \quad 0.00499 \quad 0.00049)$

t tests

$t := \overrightarrow{\frac{b}{se\_b}}$ $\quad$ $t^T = (44.3313 \quad 12.15164 \quad -9.18287 \quad 6.14094 \quad 25.55555)$

2010 to 2020 Climate Forecasting:

Empirical Component Data and Forecast: X1, X2, X3, X4

X1 = ENSO: 1980 through 2010, then mimic 1992 to 1997 twice, $\Delta t_E = 3$

X2 = Monthly Volcanic Aerosols, repeat Pinatubo eruption with a peak in 2014, $\Delta t_V = 6$,

X3 = Irradiance: 1980 through 2009, then replicate cycle 23, $\Delta t_S = 0$:

X4 = Anthropogenic, $\Delta t_A = 17*12$ (17 Yr Delay), then maintain trend.

Multi-Variate Component Forecast Model (βs Determined from Previous Regression):

$$Y_i = \beta_0 + \beta_1 \cdot X1_i + \beta_2 \cdot X2_i + \beta_3 \cdot X3_i + \beta_4 \cdot X4_i + \varepsilon_i$$

The design matrix for our Temp Stats data can be constructed with the statements

$X^{\langle 1 \rangle} := x1 \qquad X^{\langle 2 \rangle} := x2 \qquad X^{\langle 3 \rangle} := x3 \qquad X^{\langle 4 \rangle} := x4$

b := READPRN("Emp_ESVA_Coefficients.txt")

$$\Delta T_{forecast} := b_0 + b_1 \cdot x1 + b_2 \cdot x2 + b_3 \cdot x3 + b_4 \cdot x4$$

The regression and estimation forecast: