# A long-term numerical solution for the insolation quantities of the Earth

## A long-term numerical solution for the insolation quantities of the Earth

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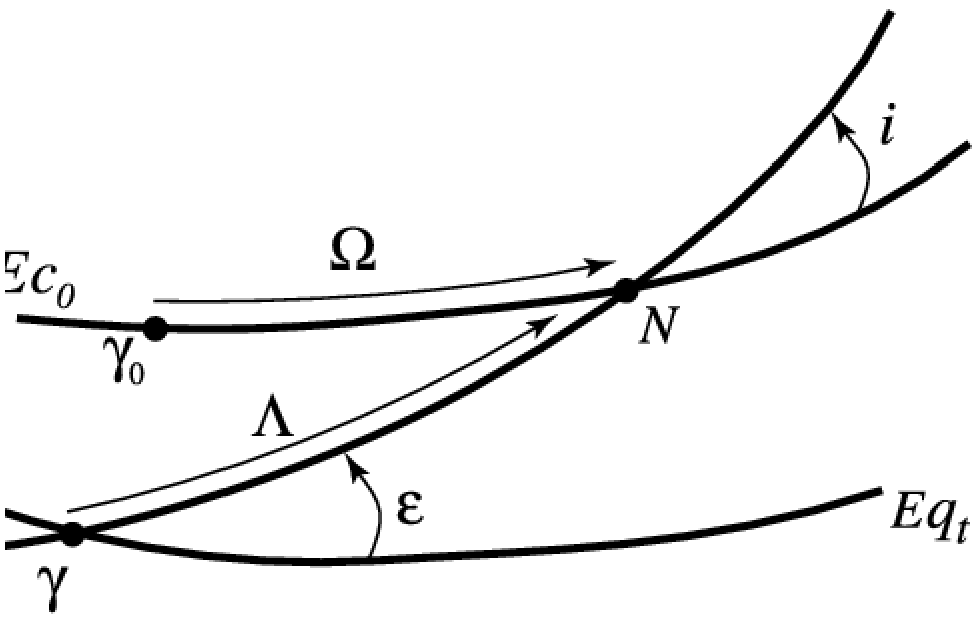
Fundamental planes for the **definition of precession and obliquity**. $Eq_t$ and $Ec_t$ are the mean equator and ecliptic at date t. $Ec_0$ is the fixed ecliptic at Julian date J2000, with equinox $\gamma_0$. The general **precession in longitude ψ** is defined by $\psi = \Lambda - \Omega$. ω is the longitude of the node, and i the inclination. The angle ε between $Eq_t$ and $Ec_t$ is the obliquity.




**Analytical approximations of Orbital motion**

It is interesting for practical use to have an analytical expression for the main orbital quantities of the Earth. From the numerical values of La2004, we have performed a **frequency analysis** (Laskar 1990, 1999, 2003) in order to obtain a **quasiperiodic approximation** of the solutions over a few Myr. In order to be consistent with the remaining part of the paper, we chose a time interval covering 20 Myr. As we are mostly interested over negative time, we made the analysis from -15 Myr to +5 Myr, as usually, the precision of the approximation decreases at the edges of the time interval. The analysis of the **eccentricity variables** z=e/exp (i ω) is given in

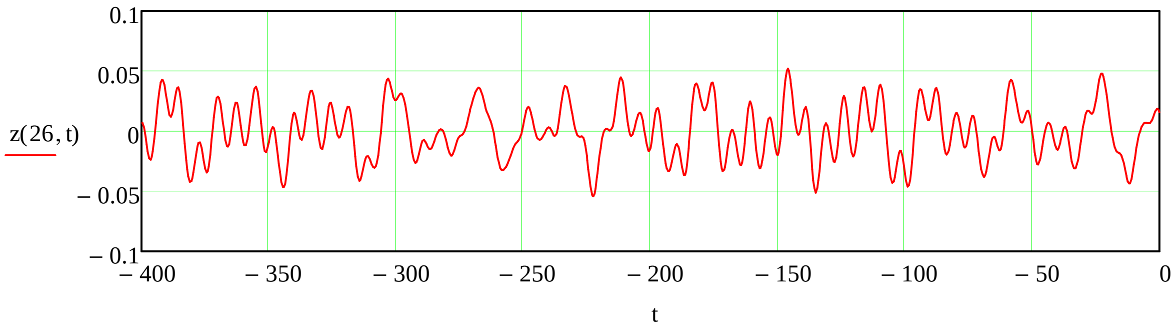
**Frequency decomposition** of z=e/exp (i ω) for the Earth on the time interval [-15,+5] Myr
n μ_k ("/yr) bk φ_k (degree)

$kk := 1 .. 23$

Table4 := READPRN("Long-term numerical solution-Table4.TXT") rows(Table4) = 26 $i := j$

$arcsec := \frac{deg}{360}$ $\mu := Table4^{\langle 1 \rangle} \cdot arcsec$ $b := Table4^{\langle 2 \rangle}$ $\varphi := Table4^{\langle 3 \rangle} \cdot deg$ $U := 10^3$

$$z(N,t) := \sin\left[\sum_{k=1}^{N} \left(b_k \cdot \sin\left(\mu_k \cdot U \cdot t + \varphi_k\right)\right)\right]$$




The solution for the **inclination variables** z=\sin i/2 \exp {i}\Omega$ where $i, \Omega$ are the Earth inclination and longitude of node with respect to the fixed ecliptic and equinox J2000, limited to 24 quasiperiodic terms

Table 5: Frequency decomposition of ζ(i,Ω) for the Earth on the time interval [-15,+5] Myr (Eq. (26))
n ν_k ("/yr) ak φk$ (degree)

Table5 := READPRN("Long-term numerical solution-Table5.TXT") $i := 23$

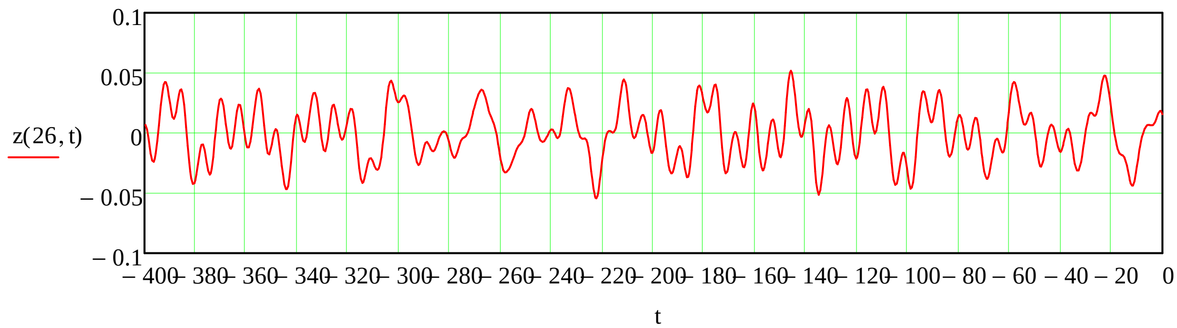
$\nu := Table5^{\langle 1 \rangle} \cdot arcsec$ $a := Table5^{\langle 2 \rangle}$ $\varphi := Table5^{\langle 3 \rangle}$

$p0 := 50.467718 \cdot arcsec$ $p1 := -13.526564 \cdot arcsec$ $\phi 0 := 171.424$ $\varepsilon 0 := 23.254500$

$$\psi(t) := 49086 + p0 \cdot t + p1 \cdot t^2 + 42246 + \frac{(\nu_0)^2}{(\nu_0 + 2 \cdot p1 \cdot t)^2} \cdot \cos\left[\left(\nu_0 \cdot t + p1 \cdot t^2 + \phi 0\right) \cdot \frac{2 \cdot \pi}{360}\right]$$

$$\zeta(i,\Omega) := \sin\left(\frac{i}{2 \cdot \exp\left(i \cdot \Omega \cdot \frac{2 \cdot \pi}{360}\right)} \cdot \frac{2 \cdot \pi}{360}\right) \qquad \zeta := \sum_{k=1}^{24} a_k$$

With the quasiperiodic approximation the solutions for precession and obliquity are obtained through the precession Eq. (6). In absence of planetary perturbations, the obliquity is constant ( ε=\ε_0), and




$\text{arcminute} := \frac{1}{60}$ $\alpha := 50.29\cdot\text{arcminute}$

We have thus p is the precession frequency. $p := \alpha\cdot\cos(\varepsilon_0)$ $\psi := \psi_0 + p\cdot t$

**Approximation for the obliquity**

As for the solutions of the orbital elements, we have obtained an approximation formula for the evolution of the obliquity of the Earth over the period from -15 Myr to +5 Myr. Obtaining this solution presents some additional difficulties because of the dissipative effects in the Earth-Moon system that induces a significative change of the precession frequency. The obliquity is obtained on the form Table 7: Approximation for the obliquity of the Earth, following Eq. (33). This expression is not strictly quasiperiodic, because of the presence of the dissipative term p1 in the evolution of the precesion frequency (33).

Table7: v' _k ("/yr) P (yr) a'k ϕ _ -k (degree) Note:prime " ' " is Ctrl F7

Table7 := READPRN("Long-term numerical solution-Table7.TXT") cols(Table7) = 4 rows(Table7) = 23

$\nu'' := \text{Table7}^{\langle 1\rangle}\cdot\text{arcsec}$ $a' := \text{Table7}^{\langle 3\rangle}$ $\phi' := \text{Table7}^{\langle 4\rangle}\cdot\text{deg}$ $p1 := -13.526564\cdot\text{arcse}$

$$\varepsilon(N,t) := \varepsilon_0 + \sum_{k=1}^{N}\left[a'_k\cdot\cos\left[\left(\nu''_k + p1\cdot t\cdot U\right)\cdot t\cdot U + \phi'_k\right]\right]$$