# XXII. Simulation of Grover's Quantum Search Algorithm

Grover's algorithm allows searching an unstructured database of N=2^n elements in O(sqrt(N)) time by amplifying the probability of the target (marked) state.

Components:
1. Superposition Initialization:
  - The initial state |> = (1/sqrtN) * |x> spans all basis states equally.
  - For 4 qubits, N = 16, so each amplitude = 1/4.

2. Oracle U_w:
  - U_w is a quantum operator that flips the phase of the target state.
  - In this simulation, it is defined as:
    U_w = I - 2|w><w|, or simply negating amplitude of |w>
  - Example: If the target is |1111> (index 15), then U_w[15,15] := -1.

3. Diffusion Operator D:
  - Reflects the state about the mean amplitude:
    D = 2|><| - I  - This "inverts about the mean" to amplify the marked state's probability.

4. Grover Iterate:
  - Each Grover iteration is:   D * U_w *
  - After r ~ floor(pi/4 * sqrt(N)) iterations, the amplitude of |w> is maximized.

Amplitude Evolution: A 2D matrix [j, i] stores the amplitude of basis state j at iteration i.

Probability is computed as:
  P[j, i] = |[j, i]|^2
For N=16, maximum amplification of the target state |1111> occurs around iteration r=3.

Analytical Prediction:
After r iterations, the amplitude of the marked state |w> is approximately:
  sin((2r + 1) * theta), where sin(theta) = 1/sqrtN

For N = 16:
  theta ~ arcsin(1/4) ~ 0.2527 radians
  After 3 iterations:
    amplitude ~ sin(7 * 0.2527) ~ 0.974
    probability ~ (0.974)^2 ~ 0.948
This matches the simulation's output and confirms correctness.

Symbol Gloss
- U_w: Oracle operator (marks the solution)
- D: Diffusion operator (amplitude inversion)
- : State vector
- P: Probability vector (squared modulus of )

**Conclusion:**
Grover's algorithm achieves quadratic speedup by constructive interference.

# Grover's Quantum Search Algorithm for 4 Qubits with Amplitude Evolution

Grover's algorithm allows searching an unstructured database of N=2^n elements in O(sqrt(N)) time by amplifying the probability of the target (marked) state.

$n := 4 \qquad N := 2^n \qquad N = 16 \qquad f(i,j) := 1 \qquad ones := matrix(16,1,f) \qquad I := Ident(16)$

**// Initial equal superposition state**

$$\psi_0 := \frac{1}{\sqrt{N}} \cdot ones$$

**// Grover Diffusion Operator Generator**

$$D := 2 \cdot \psi_0 \cdot \psi_0^T - I$$

**Function to Replace Matrix Element with Bit, B**

$$Replace(M,R,C,B) := \begin{vmatrix} M_{R,C} \leftarrow B \\ M \end{vmatrix}$$

**// Oracle Matrix: flips sign of solution state**

$U_\omega$ is a quantum operator that flips the phase of the target state(assume target is |1111⟩ = index 15). It achieves quadratic speedup by constructive interference.

$$U_\omega := Replace(I, 15, 15, -1)$$

**Number of iterations (floor of π/4 * sqrt(N))**

$$r := floor\left(\frac{\pi}{4} \cdot \sqrt{N}\right) \qquad r = 3$$

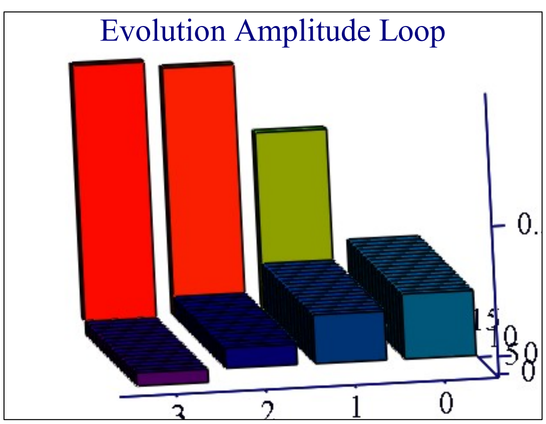
**// Evolution Amplitude Loop**

Simulate how the quantum state ψ evolves over several iterations of Grover's algorithm

$$Evol(\psi) := \begin{vmatrix} \text{for } i \in 1..r \\ \quad \begin{vmatrix} \psi i \leftarrow D \cdot U_\omega \cdot \psi^{\langle i-1 \rangle} \\ \psi^{\langle i \rangle} \leftarrow \psi i \end{vmatrix} \\ \psi \end{vmatrix}$$

$$EV := Evol(\psi_0)$$

$$EV = \begin{pmatrix} 0.25 & 0.188 & 0.078 & -0.051 \\ 0.25 & 0.188 & 0.078 & -0.051 \\ 0.25 & 0.188 & 0.078 & -0.051 \\ 0.25 & 0.188 & 0.078 & -0.051 \\ 0.25 & 0.188 & 0.078 & -0.051 \\ 0.25 & 0.188 & 0.078 & -0.051 \\ 0.25 & 0.188 & 0.078 & -0.051 \\ 0.25 & 0.188 & 0.078 & -0.051 \\ 0.25 & 0.188 & 0.078 & -0.051 \\ 0.25 & 0.188 & 0.078 & -0.051 \\ 0.25 & 0.188 & 0.078 & -0.051 \\ 0.25 & 0.188 & 0.078 & -0.051 \\ 0.25 & 0.188 & 0.078 & -0.051 \\ 0.25 & 0.188 & 0.078 & -0.051 \\ 0.25 & 0.188 & 0.078 & -0.051 \\ 0.25 & 0.688 & 0.953 & 0.98 \end{pmatrix}$$




EV