XXVI. QM: Schrödinger Wavefunction, Matrix, and Wigner Phase Space

We will model three Formulations of Quantum Mechanics:

Schrödinger Wavefunction, Matrix, and Wigner Phase Space

There are seven commonly used nonrelativistic formulations for quantum mechanics. These are the wavefunction, matrix, path integral, phase space, density matrix, second quantization, variational, formulations. Also mentioned are the many-worlds and transactional interpretations. The various formulations differ dramatically in mathematical and conceptual overview, yet each one makes identical predictions for all experimental results.

A. The matrix formulation (Heisenberg)

The matrix formulation of quantum mechanics, developed by Werner Heisenberg in June of 1925, was the first formulation to be uncovered. The wavefunction formulation, which enjoys wider currency today, was developed by Erwin Schrödinger about six months later.

B. The wavefunction formulation (Schrödinger)

Compared to the matrix formulation, the wavefunction formulation of quantum mechanics shifts the focus from "measurable quantity" to "state." The state of a system with two particles ~ ignoring spin ! is represented mathematically by a complex function in six-dimensional configuration space, namel .

C. Phase space formulation - See Section XXIV: The Wigner Quasiprobability Distribution)

For a single particle restricted to one dimension, the Wigner phase-space distribution function is

$$W(x,p,t)=\frac{1}{2\pi\hbar}\int_{-\infty}^{+\infty}\psi^*(x-\tfrac{1}{2}y,t)\ \times\psi(x+\tfrac{1}{2}y,t)e^{-ipy/\hbar}\,dy$$

D. The path integral formulation (Feynman)

The path integral formulation (also called the sum-over-histories formulation) shifts the focus from "state" to "transition probability."

E. Density matrix formulation

The density matrix corresponding to a pure state |ψ> is the outer product $\hat{\rho}=|\psi\rangle\langle\psi|$

Given the density matrix pˆ, the quantal state |ψ> can be found as follows: First select an arbitrary state |ϕ>. The unnormalized ket |ψ> is pˆ|ϕ> (as long as this quantity does not vanish).

F. Second quantization formulation This formulation features operators that create and destroy particles. It was developed in connection with quantum fixed theory, where such actions are physical effects ~ for example, an electron and a positron are destroyed and a photon is created ! .

G. Variational formulation

The "variational formulation" must not be confused with the more-commonly-encountered "variational method", which provides a bound on the ground state energy. Instead the variational formulation provides a full picture describing any state—not just the ground state—and dictating its full time evolution—not just its energy. It is akin to Hamilton's principle in classical mechanics.