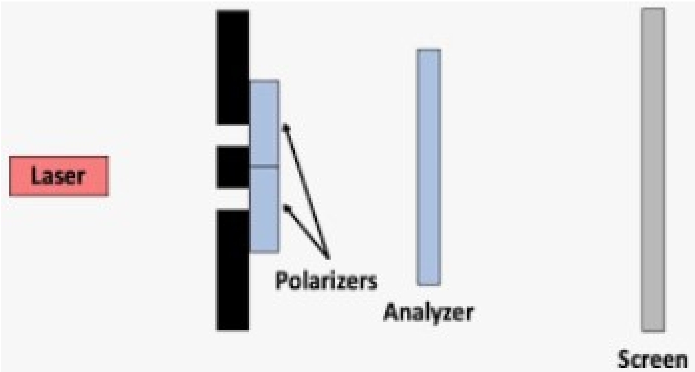
XXIX. The Quantum Eraser



Assuming (initially) infinitesimally thin slits, the photon wave function at the slit screen is an entangled superposition of being at top slit with vertical polarization and bottom slit with polarization at an angle θ relative to the vertical. This entanglement provides which-way information if θ is not equal to 0 and, therefore, has an important effect on the diffraction pattern.

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left[|x_1\rangle\,|v\rangle + |x_2\rangle|\theta\rangle\right]$$

This state is projected onto ϕ and p because a ϕ-oriented polarizer (eraser) precedes the detection screen and because a diffraction pattern is actually the momentum distribution of the scattered photons. In other words, position is measured at the slit screen and momentum is measured at the detection screen.

$$\langle p\phi|\Psi\rangle = \frac{1}{\sqrt{2}}\left[\langle p|x_1\rangle\,\langle\phi|v\rangle + \langle p|x_2\rangle\,\langle\phi|\theta\rangle\right]$$

$$\langle p\phi|\Psi\rangle = \frac{1}{\sqrt{2}}\left[\langle p|x_1\rangle\cos(\phi) + \langle p|x_2\rangle\cos(\theta-\phi)\right]$$

The polarization brackets $\langle p\phi|\Psi\rangle$ (amplitudes) are easily shown to be the **above** trigonometric functions.

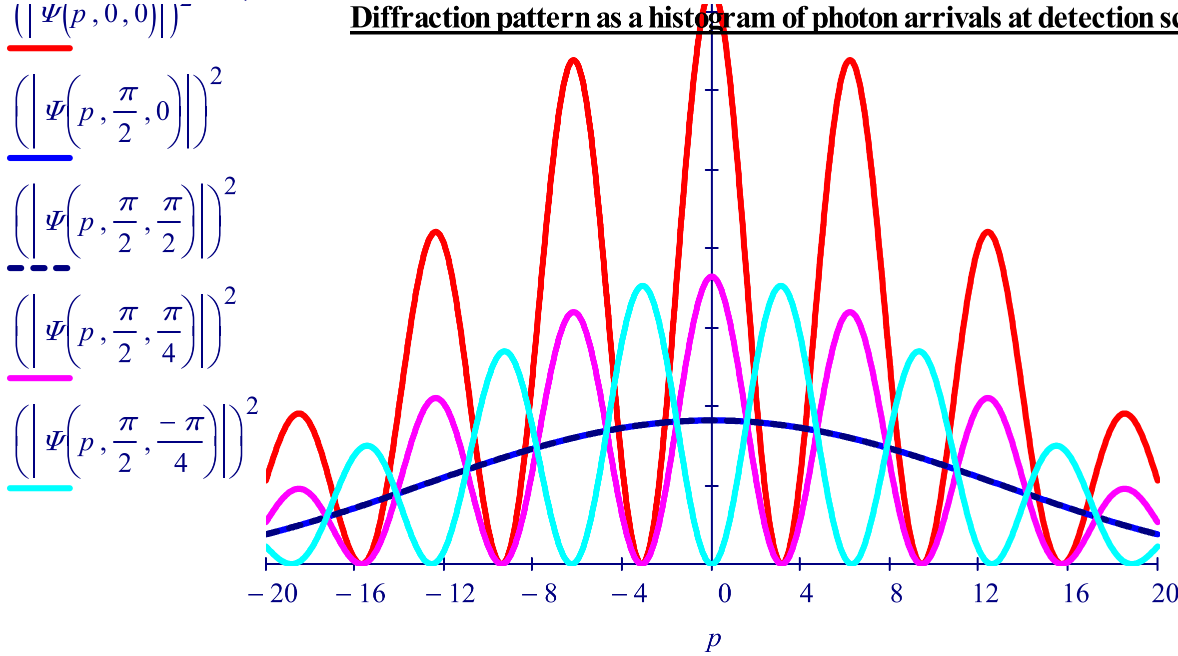
The position-momentum brackets $\langle p|x\rangle$ are the position eigenstates in the momentum representation and are given by:

$$\langle p|x\rangle = \frac{1}{\sqrt{2\pi\hbar}}\exp\left(-\frac{ipx}{\hbar}\right)$$

this allows us to write $\langle p\phi|\Psi\rangle = \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2\pi\hbar}}\exp\left(-\frac{ipx_1}{\hbar}\right)\cos(\phi) + \frac{1}{\sqrt{2\pi\hbar}}\exp\left(-\frac{ipx_2}{\hbar}\right)\cos(\theta-\phi)\right]$

Working in atomic units (h = 2π) and now assuming slits of finite width this expression becomes,

$$\Psi(p,\theta,\phi) := \frac{1}{\sqrt{2}}\left(\int_{x_1-\frac{\delta}{2}}^{x_1+\frac{\delta}{2}} \frac{1}{\sqrt{2\cdot\pi}}\cdot exp(-i\cdot p\cdot x)\cdot\frac{1}{\sqrt{\delta}}\,dx\cdot cos(\phi) + \int_{x_2-\frac{\delta}{2}}^{x_2+\frac{\delta}{2}} \frac{1}{\sqrt{2\cdot\pi}}\cdot exp(-i\cdot p\cdot x)\cdot\frac{1}{\sqrt{\delta}}\,dx\cdot cos(\theta-\phi)\right)$$



Diffraction pattern as a histogram of photon arrivals at detection screen

Discussion of Results

The polarizer at top slit is always oriented vertically so only the orientations (θ and ϕ) of the other polarizers need to be specified.

The photons emerging from the slits are vertically polarized and encounter a vertical polarizer before the detection screen. This is the plot of $(|\Psi(p,0,0)|)^2$. There is no which-way information in this experiment and 100% of the photons emerging from the vertically polarized slit screen reach the detection screen.

$$\int_{-\infty}^{\infty} (|\Psi(p,0,0)|)^2 dp \text{ float}, 3 \rightarrow 1.00$$

The crossed polarizers at the slit screen provide which-way information and the interference fringes disappear if the third polarizer is vertically or horizontally oriented. This is shown by the plots of $(|\Psi(p,\pi/2,0)|)^2$ and $(|\Psi(p,\pi/2,\pi/2)|)^2$. Furthermore, relative to the reference experiment, 50% of the photons reach the detection screen.

$$\int_{-\infty}^{\infty} \left(\left|\Psi\left(p,\frac{\pi}{2},0\right)\right|\right)^2 dp \text{ float}, 3 \rightarrow 0.500$$

$$\int_{-\infty}^{\infty} \left(\left|\Psi\left(p,\frac{\pi}{2},\frac{\pi}{2}\right)\right|\right)^2 dp \text{ float}, 3 \rightarrow 0.500$$

In the absence of the third polarizer, there is also no diffraction pattern but 100% of the photons reach the detection screen. [θ=π/2,ϕ=π/4] and [θ=π/2,ϕ=−π/4]

The which-way information provided by the crossed polarizers at the slit screen is erased by diagonally and anti-diagonally oriented polarizers in front of the detection screen. This is shown by the plots of $(|\Psi(p,\pi/2,\pi/4)|)^2$ and $(|\Psi(p,\pi/2,-\pi/4)|)^2$

The reason the which-way information has been erased is that vertically and horizontally polarized photons emerging from slits 1 and 2 both have a 50% chance of passing the diagonally or anti-diagonally oriented third polarizer. Thus, it is impossible to determine the origin of a photon that passes the third polarizer and the interference fringes are restored. Again, for this experiment 50% of the photons reach the detection screen.

$$\int_{-\infty}^{\infty} \left(\left|\Psi\left(p,\frac{\pi}{2},\frac{\pi}{4}\right)\right|\right)^2 dp \text{ float}, 3 \rightarrow 0.500$$

$$\int_{-\infty}^{\infty} \left(\left|\Psi\left(p,\frac{\pi}{2},-\frac{\pi}{4}\right)\right|\right)^2 dp \text{ float}, 3 \rightarrow 0.500$$

The shift in the interference fringes calculated for $(|\Psi(p,\pi/2,\pi/4)|)^2$ and $(|\Psi(p,\pi/2,-\pi/4)|)^2$ is observed in the Kwiat/Hillmer experiment. The visibility of the restored fringes is maximized for $\phi = \pm\pi 4$.