

XXX. The Difference Between Fermions and Bosons: Math

Calculate the average separation, $|x_1 - x_2|$, for two fermions and two bosons in a 1D box of unit length.

$$n_1 := 1 \quad n_2 := 2 \quad \psi(x) := \sqrt{2} \cdot \sin(n_1 \cdot \pi \cdot x) \quad \Phi(x) := \sqrt{2} \cdot \sin(n_2 \cdot \pi \cdot x)$$

Fermions have antisymmetric wave functions:

$$\Psi_f(x_1, x_2) := \frac{\Psi(x_1) \cdot \Phi(x_2) - \Psi(x_2) \cdot \Phi(x_1)}{\sqrt{2}}$$

The average particle separation for indistinguishable fermions:

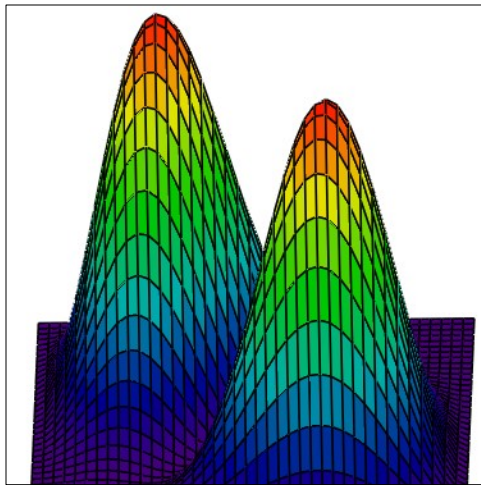
$$\text{Separation_Fermions} := \int_0^1 \int_0^1 \Psi_f(x_1, x_2) \cdot |x_1 - x_2| \cdot \Psi_f(x_1, x_2) dx_1 dx_2$$

$$\text{Separation_Fermions} = 0.383$$

The particles are correlated so as to keep them apart.

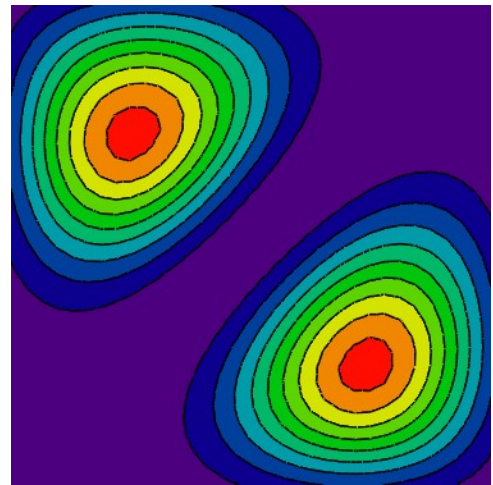
$$N := 40 \quad i := 0..N \quad x_{1i} := \frac{i}{N} \quad j := 0..N \quad x_{2j} := \frac{j}{N} \quad \psi_{i,j} := \Psi_f(x_{1i}, x_{2j})^2$$

Fermion 3D Surface Plot



Ψ_f

Fermion Contour Plot



Ψ_f

Bosons have symmetric wave functions:

$$\Psi_b(x_1, x_2) := \frac{\Psi(x_1) \cdot \Phi(x_2) + \Psi(x_2) \cdot \Phi(x_1)}{\sqrt{2}}$$

The average particle separation for indistinguishable bosons:

$$\text{Separation_Bosons} := \int_0^1 \int_0^1 \Psi_b(x_1, x_2) \cdot |x_1 - x_2| \cdot \Psi_b(x_1, x_2) dx_1 dx_2$$

Bosons Clump, Fermions Separate

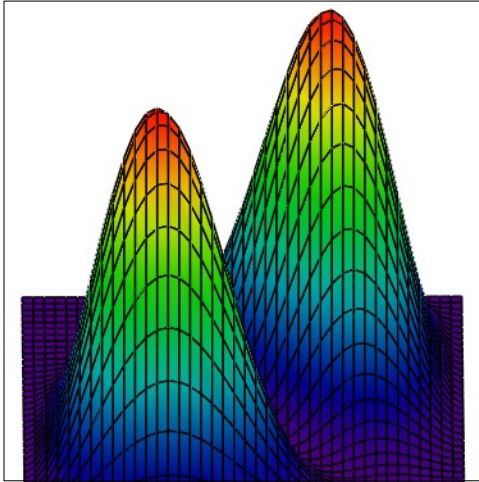
$$\text{Separation_Fermions} = 0.383$$

$$\text{Separation_Bosons} = 0.157$$

The particles are correlated so as to bring them closer together.

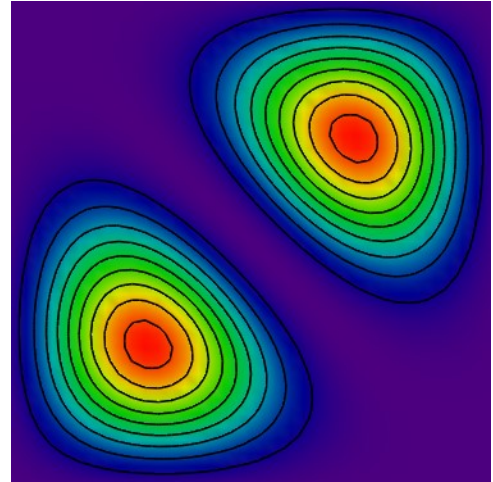
$$N := 40 \quad i := 0..N \quad x_{1_i} := \frac{i}{N} \quad j := 0..N \quad x_{2_j} := \frac{j}{N} \quad \psi_{i,j} := \psi_b(x_{1_i}, x_{2_j})^2$$

Boson 3D Surface Plot



ψ_b

Boson Contour Plot



ψ_b

All fundamental particles (electrons, neutrons, protons, photons, etc.) are either bosons or fermions. Composite entities such as the elements also fall into these two categories. The fundamental distinction is spin: bosons have integer spin (0, 1, 2, ...) while fermions have half-integer spin (1/2, 3/2, ...).

The dramatic difference in behavior between bosons and fermions has led to a sociology of fundamental particles. Bosons are social and gregarious, while fermions are antisocial and aloof.