VII. Quantum Superposition

Tutorial: Wikipedia

Quantum superposition is a fundamental principle of quantum mechanics. In classical mechanics, things like position or momentum are always well-defined. We may not know what they are at any given time, but that is an issue of our understanding and not the physical system. In quantum mechanics, a particle can be **in a superposition of different states**. It can be in **two places at once** (see double-slit experiment). A measurement always finds it in one state, but before and after the measurement, it interacts in ways that **can only be explained by having a superposition of different states.**

Mathematically, much like waves in classical physics, any two (or more) quantum states can be added together ("superposed") and the result will be another valid quantum state; conversely, **every quantum state can be represented as a sum of two or more other distinct states.** Mathematically, it refers to a property of solutions to the Schrödinger equation; **since the Schrödinger equation is linear, any linear combination of solutions will also be a solution(s).**

An example of a physically observable manifestation of the wave nature of quantum systems is the interference peaks from an electron beam in a double-slit experiment. The pattern is very similar to the one obtained by diffraction of classical waves.

Another example is a quantum logical qubit state, as used in quantum information processing, **which is a quantum superposition of the "basis states" $|0\rangle$ and $|1\rangle$.** Here $|0\rangle$ is the Dirac notation for the quantum state that will always give the result **0** when converted to classical logic by a measurement. Likewise |1> is the state that will always convert to **1**. Contrary to a classical bit that can only be in the state corresponding to 0 or the state corresponding to 1, **a qubit may be in a superposition of both states**. This means that the probabilities of measuring 0 or 1 for a qubit are in general neither 0.0 nor 1.0, and multiple measurements made on qubits in identical states will not always give the same result.

Concept

The principle of quantum superposition states that if a physical system may be in many configurations -arrangements of particles or fields—then the **most general state is a combination of all of these possibilities**, where the amount in each configuration is specified by a complex number.

For example, if there are two configurations labeled by 0 and 1, the most general state would be where the coefficients are complex numbers describing how much goes into each configuration.

$$c_0|0\rangle + c_1|1\rangle$$

The principle was described by Paul Dirac as follows:

The general principle of superposition of quantum mechanics applies to the states [that are theoretically possible without mutual interference or contradiction] ... of any one dynamical system. **The original state must be regarded** as the **result of a kind of superposition of the two or more new states**, in a way that cannot be conceived on classical ideas. **Any state** may be considered as the **result of a superposition of two or more other** states, and indeed in an **infinite number of ways**. Conversely, any two or more states may be superposed to give a **new state**. For an equation describing a physical phenomenon, the superposition principle states that a combination of solutions to a linear equation is also a solution of it. When true, the equation is said to obey the superposition principle.

Pure States: The mathematical formulation of quantum mechanics, pure quantum states correspond to **vectors in a Hilbert space**, while **each observable** quantity (such as the energy or momentum of a particle) is **associated with a mathematical operator.** The operator serves as a linear function which acts on the states of the system. **The eigenvalues** of the operator correspond to the **possible values of the observable**.

The superposition principle and the wave function

Exploring the Quantum, Haroche and Raim

The superposition principle and the wave function Let us start by recalling briefly the general framework of quantum theory. **Each state** of a microscopic system A is represented by **a vector in an abstract Hilbert space H_A** and the **physical observables** of this system are associated to Hermitian (self-adjoint) **operators in H_A .** The linear combination and scalar product of state vectors as well as the operator algebra in H_A are defined in all quantum mechanics textbooks.

The description of the most general state |ψ> of *A* requires the **definition of a reference basis {| i >}** in H_A, **obeying the orthogonality** and closure relationships:

$$\langle i | j \rangle = \delta_{ij} \; ; \qquad \sum_i |i\rangle \langle i| = \mathbb{1}$$

where δ_{ij} is the usual Kroneker symbol and **1** the unity operator in H_A . The **basis states** are the **eigenstates of a complete ensemble of commuting observables** $O_1, O_2, \ldots, O_k$ which define a 'representation' in H_A .

Once the representation basis is known, any state |ψ> of A is developed as:

$$|\psi\rangle = \mathbb{1} |\psi\rangle = \sum_i |i\rangle \langle i | \psi\rangle$$

a **linear combination of basis states**, entirely defined by the list of *C*-number coefficients c_i = < i |ψ>. A measurement of the complete ensemble $\{O_k\}$ r**andomly projects |ψ>** into one of **the | i | ψ> states** with the probability $p_i = |c_i|^2$. The <i|ψ> scalar product coefficients are thus called 'probability amplitudes'. The normalization of the state (|<ψ|ψ> = 1) ensures that the total probability of all measurement outcomes is equal to 1. Immediately after the **measurement**, the system's state is irreversibly changed,

'jumping' from |ψ> into one of the | i>'s.

Repeating the **measurement immediately afterwards** (i.e. before the system has had time to evolve) leaves *A* with **unit probability** in the **same |i> state**. At this stage, we just enunciate the postulates of the quantum theory of measurement. A description of measurement procedures, which includes a definition of a measuring apparatus and of its coupling with *A*. We will then try to sharpen our understanding of the irreversible evolution of a quantum system upon measurement, certainly the most difficult aspect of quantum theory.

In everyday language, the above equation, can be loosely expressed by saying that

if a system can exist in different configurations
(corresponding for example to different classical descriptions),
it can also exist in a superposition of these configurations,

so to speak 'suspended' between them. This layman's language is imprecise though, while the above mathematical formula is unambiguous.

***Fundamentals of Quantum Entanglement*, F J Duarte**

The physics path (To Quantum Entanglement)

The physics path is a pragmatic, measurement driven, avenue to quantum entanglement. It began with a paper by Dirac on **pair production** (Dirac 1930) and some sixteen years later was followed by a transparent and profound statement by John Wheeler that captures the very essence of quantum entanglement: ‘if one of these photons is linearly polarized in one plane, then the photon that **goes off in the opposite direction with equal momentum** is **linearly polarized** in the perpendicular plane ’ (Wheeler 1946-). Wheeler made his statement in reference to a positron–electron annihilation process, $e^+e^- \rightarrow \gamma_1\gamma_2$, that leads to the emission of two quanta in opposite directions. Ward followed with a disclosure of the derivation of the quantum entanglement probability amplitude

$$|\psi\rangle = (|x_1, y_2\rangle - |y_1, x_2\rangle)$$

Mathematically, this means Entanglement is a Sum of Products of Wave-functions.

It should be noted that this probability amplitude is essential for the correct derivation of the final quantum probability for the quantum scattering equation published by Pryce and Ward (1947).
It should also be stated categorically that the above $|\psi\rangle$ expression for the probability amplitude includes and contains all the physics relevant to quantum entanglement experiments. All this physics was done pre-Bell theorem in a complete vacuum of philosophical discussions and in the total absence of concern, or preoccupation, with hidden variable theories.
What should be kept in mind is that even though today all of the developments in the field of quantum entanglement revolve around the probability amplitude for quantum entanglement

$$|\psi\rangle = (|x_1, y_2\rangle - |y_1, x_2\rangle)$$

there is almost no acknowledgement of its origin or the physics path that led to its discovery. This monograph is designed to provide a perspective on quantum entanglement from the philosophical and the physics perspectives by including all the relevant literature. This approach removes the ‘cloud of mystery’ that surround quantum entanglement.

The field of quantum entanglement

The emergence of the combined words quantum entanglement, in the open literature, appears to go back to the mid-late 1980s (Ghirardi et al 1987). This was a few years after the optical experiments on quantum entanglement by Aspect et al (1981, 1982a,1982b).

Today the fireld of quantum entanglement is enormous and it is divided roughly into three main sub fields:

- quantum cryptography,
- quantum teleportation,
- and quantum computing.
- Quantum communications is part of quantum cryptography.

On paper, judging by citations, these sub fields have been heavily influenced by the ideas and concepts derived from the philosophical path to quantum entanglement.
On the other hand, also judging from citations,
the acknowledgement of the physics path has been utterly minuscule.
This almost non-existing recognition has persisted albeit the all-important probability amplitude for quantum entanglement, which was discovered in a vacuum of philosophical arguments: