Properties of Mixed States

Mixed State:
Represents incomplete knowledge about the system.
Described by a density matrix ρ that is a **weighted sum** of **pure state projectors**:

$$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|, \quad \text{where } \sum_i p_i = 1, \quad 0 \le p_i \le 1$$

Tr(ρ²) < 1 (unless it happens to be a pure state in disguise).

$tr(\blacksquare)$

A Mixed State is a statistical ensemble of pure states. For example:

ρ = 1/2 |00⟩⟨00| + 1/2 |11⟩⟨11|
This means the system is 50% likely in ***|00⟩*** and 50% likely in ***|11⟩***,
but never in coherent superposition.

Mixed State Example (2 Qubits)
Let's say we have a quantum system that is
In the Bell state:

- In the Bell state $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ with 70% probability
- In the separable state $|01\rangle$ with 30% probability

Step 1: Define Pure States as 4×1 Vectors

$$psi_{plus} := \frac{1}{\sqrt{2}} \cdot \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} \qquad psi_{plus} := BSG \cdot OO \qquad psi_{plus} = \begin{pmatrix} 0.707 \\ 0 \\ 0 \\ 0.707 \end{pmatrix} \qquad O1 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

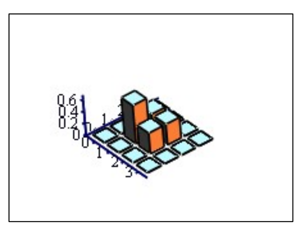
Density Matrices of Each State

$$\rho_{plus} := psi_{plus} \cdot psi_{plus}^{T} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \cdot \frac{1}{2} \qquad \rho_{01} := O1 \cdot O1^{T} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

The Mixed State

$$\rho := 0.7 \cdot \rho_{plus} + 0.3 \cdot \rho_{01}$$

$$\rho = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.65 & 0.35 & 0 \\ 0 & 0.35 & 0.35 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$



$$tr(\rho^2) = 0.79$$

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