Difference Between Bloch States and Bell States

Bloch states and Bell states are related but distinct concepts in quantum mechanics.
Bloch states describe the wavefunctions of particles in a periodic potential, while
Bell states are a specific set of maximally entangled states of two qubits.

Bell states:
A set of four maximally entangled states of two qubits (the smallest unit of quantum information). It's a state where the two qubits are so intrinsically linked that the state of one is instantly known when the state of the other is measured, regardless of the physical distance between them.

They are named after John Bell, who developed Bell's theorem, a key result in quantum mechanics. Bell states are fundamental in quantum information theory and quantum communication protocols like quantum teleportation.
They exhibit perfect correlations between the two qubits, even when separated by a large distance, which is a key aspect of entanglement.

Bloch states:
A type of wavefunction used to describe a particle (like an electron) moving in a periodic potential, such as a crystal lattice. They are characterized by a periodicity that matches the periodicity of the potential, and are often described using a wavevector (k) and a periodic part of the wavefunction (u(r)).
Bloch states are important for understanding the behavior of electrons in solids and other periodic systems.
States in the Bloch sphere are of the form $|\psi\rangle = \cos(\theta/2)\,|0\rangle + e^{i\phi}\sin(\theta/2)\,|1\rangle$,
The Bloch sphere is a geometrical representation of the pure state space of a two-level QM system (qubit).
The Bloch sphere is a unit 2-sphere, with antipodal points corresponding to a pair of mutually orthogonal state vectors.
The north and south poles of the Bloch sphere are typically chosen to correspond to the standard basis vectors $|0\rangle$ and $|1\rangle$, respectively, which in turn might correspond e.g. to the spin-up and spin-down states of an electron. This choice is arbitrary, however. Given an orthonormal basis, any pure state $|\phi\rangle$ of a two-level quantum system can be written as a superposition of the basis vectors $|0\rangle$ and $|1\rangle$, where the coefficient of (or contribution from) each of the two basis vectors is a complex number

Difference Block and Bell States
You cannot plot the Bell state using a single Bloch sphere, as it can be used exclusively for a single qubit representation.

Some Commonly Used Symbols in Quantum Mechanics

Notation	Description
c^*	Complex conjugate of a complex number c
(example)	$c^* = (a+bi)^* = a - ib$ (a and b are real numbers)
$\|\psi\rangle$	State vector in a Hilbert space. a ket vector
$\langle\psi\| = (\|\psi\rangle)^\dagger$	State vector in a Hilbert space. a bra vector
$\langle\varphi\|\psi\rangle$	Scalar product of state vectors $\|\psi\rangle$ and $\|\varphi\rangle$
$\|\Psi\rangle = c_1\|\psi\rangle + c_2\|\varphi\rangle$	Superposition of $\|\psi\rangle$ and $\|\varphi\rangle$
$\|\psi\rangle \otimes \|\varphi\rangle$	Tensor product of $\|\psi\rangle$ and $\|\varphi\rangle$
(example)	$\begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\0\\0\\0\end{pmatrix}, \quad \begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}0\\1\\0\\0\end{pmatrix}$
A	An operator
$A^\dagger$	Hermitian conjugate of an operator A

The Superposition Principle - Photons - Polarization States, Entanglement

According to the principles of quantum mechanics, systems are set to a definite state only once they are measured. Before a measurement, systems are in an **indeterminate state**; after we measure them, they are in a definite state. If we have a system that can take on one of two discrete states when measured, we can represent the two states in Dirac notation as $|0\rangle$ and $|1\rangle$. We can then represent a **Superposition of States** as a linear combination of these states, such as

$$\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

Superposition is the linear combination of two or more state vectors is another state vector in the same Hilbert space and describes another state of the system.

As an example, let us consider a property of light that **illustrates a superposition of states**. Light has an intrinsic property called polarization which we can use to illustrate a superposition of states. In almost all of the light we see in everyday life - from the sun, for example - there is no preferred direction for the polarization. **Polarization states can be selected** by means of a polarizing filter, a thin film with an axis that only allows light with polarization parallel to that axis to pass through. With a single polarizing filter, we can **select one** polarization of light, for example *vertical polarization*, which we can denote as $|\uparrow\rangle$. Horizontal polarization, which we can denote as $|\rightarrow\rangle$, is an orthogonal state to vertical polarization. Together, **these states form a basis** for any polarization of light. That is, any polarization state $|\psi\rangle$ can be written as linear combination of these states. We use the Greek letter ψ to denote the state of the system

$$|\psi\rangle = \alpha\,|\uparrow\rangle + \beta\,|\rightarrow\rangle$$

The coefficients α and β are complex numbers known as amplitudes. The coefficient α is associated with vertical polarization and the coefficient β is associated with horizontal polarization. These have an important interpretation in quantum mechanics which we will see in Section VIII.

After selecting vertical polarization with a polarizing filter, we can then introduce a **second polarizing filter** after the first. Do you see light? If you answered no to this question, you would be correct. The horizontal state $|\rightarrow\rangle$ is orthogonal to the first, so there is no amount of horizontal polarization after the first vertical filter. Suppose now we oriented the axis of the second polarizing filter at 45°(i.e., along the diagonal ↗ between vertical ↑ and horizontal →) to the first instead of horizontally. Now we ask the same question—would we see any light get through the second filter? If you answered no to this question, you may be surprised to find the answer is yes. We would, in fact, see some amount of light get through the second filter. How could this be if all light after the first filter has vertical polarization? The reason is that **we can express vertical polarization as a superposition of two diagonal components.** That is, letting ↗ denote +45° polarization and ↖ denote -45° polarization, we may write

$$|\uparrow\rangle = \frac{1}{\sqrt{2}}|\nearrow\rangle + \frac{1}{\sqrt{2}}|\nwarrow\rangle$$

It is for this reason that we see some amount of light get past the second filter. Namely, the vertical polarization can be written as a **superposition of states**, one of which is precisely the 45° diagonal state ↗ we are allowing through the second filter. **Since the ↗ state is only one term in the superposition, not all** of the light gets through the filter, but some does. **The amount that gets transmitted is precisely 1/2 in this case.**