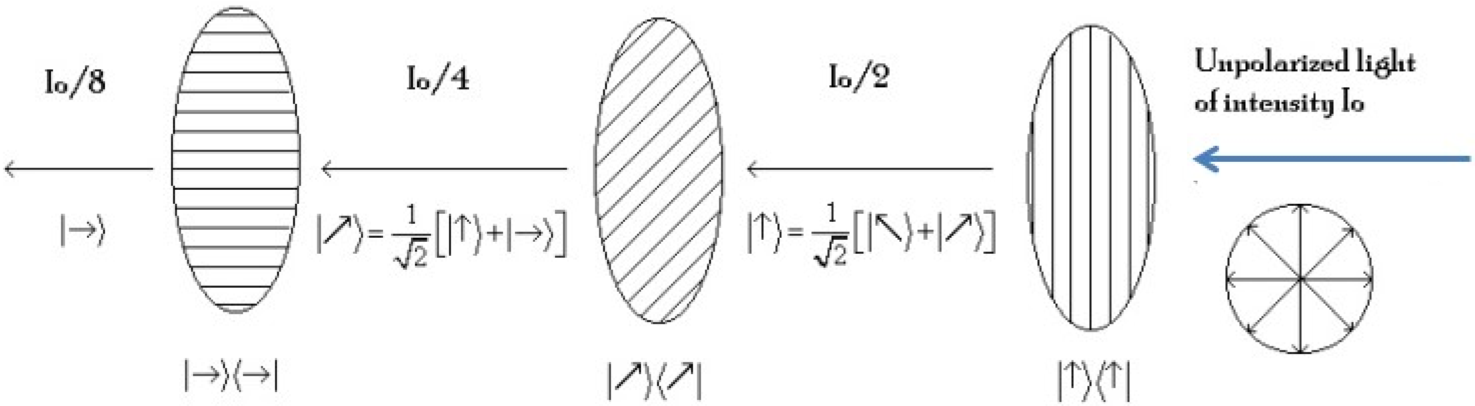
IX. Exploring the Three Polarizer "Paradox"

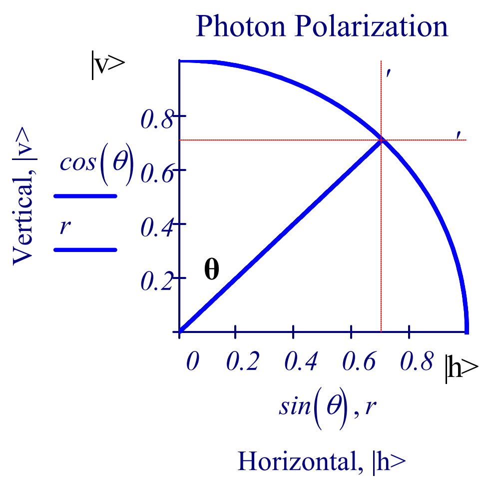
LibreTexts Quantum Tutorials



Quantum Mechanical Analysis (Refer to the Above Drawing)

A photon polarized at an **angle θ** with respect to the **Vertical** can be written as a **linear combination** (superposition) of a **vertically** polarized photon, |v>, and a **horizontally** polarized photon, |h>,. We say |v>, and |h>, are the polarization basis states, which means <v|v>, = <h|h> = 1 and <v|h> = <h|v> = 0.

$$|\Theta\rangle = |v\rangle\langle v|\Theta\rangle + |h\rangle\langle h|\Theta\rangle$$



From this figure we see that the projections of $|\Theta\rangle$ onto $|v\rangle$ ($\langle v|\Theta\rangle$) and $|h\rangle$ ($\langle h|\Theta\rangle$) are cos (Θ) and sin (Θ), respectively.

$$|\Theta\rangle = |v\rangle\cos(\Theta) + |h\rangle\sin(\Theta)$$

The probability that a photon polarized at an angle Θ will pass a vertical polarizer is

$$|\langle v|\Theta\rangle|^2 = \cos^2(\Theta)$$

The light incident on the first polarizer is unpolarized, but the photons that pass the vertical polarizer are vertically polarized. In other words **the photons are eigenfunctions of the measurement operato**r, which in this case is a vertically oriented linear polarizer. At this point only two experiments have definite outcomes.

1. The probability that vertically polarized photons will pass a second filter that is also vertically polarized is one. It is certain that a vertically polarized (Θ = 0) photon will pass a vertically polarized filter.

$$|\langle v|v\rangle|^2 = \cos^2(0^0) = 1$$

2. The probability that vertically polarized photons will pass a second filter that is horizontally polarized is zero. It is certain that a vertically polarized (Θ = π/2) photon will not pass a horizontally polarized filter.

$$|\langle h|v\rangle|^2 = \cos^2(90^0) = 0$$

For **<u>any other</u> orientation** of the second filter, quantum mechanics can only predict the probability that a **vertically polarized photon will pass**, and that probability is, of course,

$$|\langle \Theta \,|v\rangle|^2 = \cos^2(\Theta^0)$$

Now a vertically polarized photon may be written as a linear superposition of any other orthogonal basis states, for example |45°>, and |-45°>,.

$$|v\rangle = |45^0\rangle\langle 45^0\,|v\rangle + |-45^0\rangle\langle -45^0\,|v\rangle$$
$$|v\rangle = |45^0\rangle\cos(45^0) + |-45^0\rangle\cos(45^0$$
$$|v\rangle = |45^0\rangle.707 + |-45^0\rangle.707$$

If a 45° polarizer is inserted between the vertical and horizontal polarizers photons get through the horizontal polarizer that stopped them previously. Here is the quantum mechanical explanation. The probability $|\langle \Theta \,|\, v\rangle|^2$ that a vertically polarized photon will get through a polarizer oriented at an angle of 45° is, by above eqns, 1/2.

$$\left|\langle 45^0\,|v\rangle\right|^2 = \cos^2(45^0) = \frac{1}{2}$$

At this point the photon is in the state |45°>, and according to the superposition principle a photon in this state can be written as a **linear combination of |v> and |h>**,

$$|45^0\rangle = |v\rangle\langle v|45^0\rangle + |h\rangle\langle h|45^0\rangle$$
$$|45^0\rangle = |v\rangle\cos(45^0) + |h\rangle\sin(45^0$$

Therefore, the probability that this photon will pass the **<u>FINAL</u> horizontally (*h*) oriented** polarizer is

$$\left|\langle h|45^0\rangle\right|^2 = \sin^2(45^0) = \frac{1}{2} \qquad \sin(45deg)^2 = 0.5$$

Alternatively, the probability that a photon emerging from the **vertical polarizer** will pass through the **<u>FINAL</u>** horizontal polarizer in the presence of an intermediate 45° polarizer can be calculated as follows:

$$\left|\langle h\,|45^0\rangle\langle 45^0\,|v\rangle\right|^2 = \left|\sin(45^0)\cos(45^0)\right|^2 = \frac{1}{4} \qquad (\sin(45deg)\cdot\sin(45deg))^2 = 0.25$$

In other words, half of the photons that emerge from the vertical polarizer pass the 45° polarizer, and half of those pass the final horizontal polarizer. So 25% of the photons that pass the initial vertical polarizer also pass the final horizontal polarizer.

<u>The purpose of the following tutorial is to analyze the Stern-Gerlach experiment using matrix mechanics.</u>