XVII. Simulate: "Qubit Quantum Mechanics with Correlated-Photon Experiments"

Matrix Based Formalism to Model Some Classic QM Experiments, Paper by Galvez, AJP 78, 510-519 (2010)

This document uses the Mathcad programming environment to model and reproduce most of the results for the experiments presented in Professor Galvez's paper.

Optical components such as mirrors,beam splitters, wave plates, and the entire interferometer can be represented by matrices. They perform the evolution of the state of the light as it propagates.

Polarization Space and Direction of Propagation: State Vectors

Photon moving horizontally: $x := \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ Photon moving vertically: $y := \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ Null vector: $n := \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

Horizontal polarization: $h := \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ Vertical polarization: $v := \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ Diagonal polarization: $d := \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$

Single mode operators:

Projection operators for motion in the x- and y-directions: $X := \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ $Y := \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$

Operator for polarizing film oriented at angle of θ to the horizontal. $\Theta_{op}(\theta) := \begin{pmatrix} cos(\theta) \\ sin(\theta) \end{pmatrix} \cdot (cos(\theta) \;\; sin(\theta)) \rightarrow \begin{pmatrix} cos(\theta)^2 & cos(\theta)\cdot sin(\theta) \\ cos(\theta)\cdot sin(\theta) & sin(\theta)^2 \end{pmatrix}$

Symmetric non-polarizing Beam Splitter, BS

Beam splitter: $BS := \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$ Mirror: $M := \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ Phase shift: $A(\delta) := \begin{pmatrix} e^{i\cdot\delta} & 0 \\ 0 & 1 \end{pmatrix}$

Half (W2) and quarter (W4) wave plates: $W_2 := \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ $W_4 := \begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix}$ Identity: $I := \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Equa

Rotated half wave plate: $W_2(\theta) := \begin{pmatrix} cos(2\cdot\theta) & sin(2\cdot\theta) \\ sin(2\cdot\theta) & -cos(2\cdot\theta) \end{pmatrix}$ $W(\theta) := \begin{pmatrix} cos(2\cdot\theta) & sin(2\cdot\theta) & 0 & 0 \\ sin(2\cdot\theta) & -cos(2\cdot\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$

Mach-Zehnder Interferometer, MZ: $MZ(\delta) := BS\cdot A(\delta)\cdot M\cdot BS$

Two mode states and operators:

Single-photon direction of propagation and polarization states Eq 15:

$xh := \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ $xv := \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$ $yh := \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$ $yv := \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$

Two-photon direction of propagation states.

$$xx := \begin{pmatrix}1\\0\\0\\0\end{pmatrix} \quad xy := \begin{pmatrix}0\\1\\0\\0\end{pmatrix} \quad yx := \begin{pmatrix}0\\0\\1\\0\end{pmatrix} \quad yy := \begin{pmatrix}0\\0\\0\\1\end{pmatrix}$$

Two-photon polarization states.

$$hh := \begin{pmatrix}1\\0\\0\\0\end{pmatrix} \quad hv := \begin{pmatrix}0\\1\\0\\0\end{pmatrix} \quad vh := \begin{pmatrix}0\\0\\1\\0\end{pmatrix} \quad vv := \begin{pmatrix}0\\0\\0\\1\end{pmatrix}$$

Polarizing Beam Splitter, PBS, which transmits horizontally polarized photons and reflects vertically polarized photons. Eq. 24

$$PBS := xh\cdot xh^T + yv\cdot xv^T + yh\cdot yh^T + xv\cdot yv^T \qquad PBS = \begin{pmatrix}1&0&0&0\\0&0&0&1\\0&0&1&0\\0&1&0&0\end{pmatrix}$$

Kronecker is Mathcad's command for tensor multiplication, $\otimes$, of square matrices.
kronecker(M, N) Multiplies matrix N by each element of matrix M,
returning an M•N by M•N array. Arguments: M and N are square matrices.

Polarization M-Z interferometer: $MZ_P(\delta) := PBS\cdot kronecker(A(\delta), I)\cdot kronecker(M, I)\cdot PBS$

Mathcad Simulation of MZI

Mach-Zehnder interferometer for direction of propagation and polarization, which places a rotatable half-wave plate in the upper path. Eq. 24

$$MZ_{dp}(\theta, \delta) := kronecker(BS, I)\cdot kronecker(A(\delta), I)\cdot W(\theta)\cdot kronecker(M, I)\cdot kronecker(BS, I)$$

Mach-Zehnder two-photon direction-of-propagation interferometer.

$$AA(\delta) := kronecker(A(\delta), A(\delta))$$

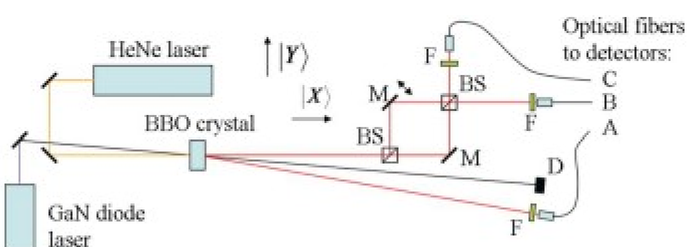
$$BSBS := kronecker(BS, BS) \qquad MM := kronecker(M, M)$$

$$MZ_{dd}(\delta) := BSBS\cdot AA(\delta)\cdot MM\cdot BSBS$$

The results of Single Photons going through the interferometer and being detected at the two outputs of the interferometer are shown in Fig. 2.

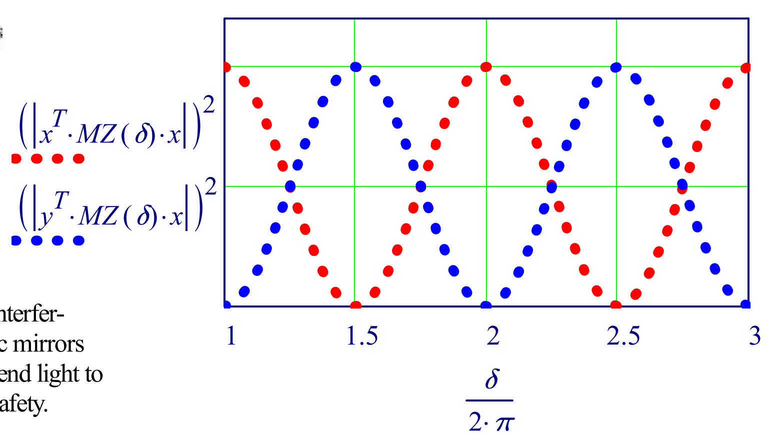
Confirm the results in Figure 2 for the Mach-Zehnder interferometer:

$\delta := 0, .125\cdot\pi .. 6\pi$



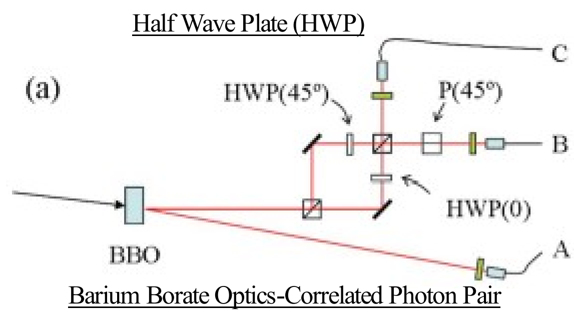
Barium Borate Optics-Correlated Photon Pair

Fig. 1. Standard layout for doing experiments with correlated photons. Interferometer components are **nonpolarizing beam splitters (BS)** and metallic mirrors (m). Band-pass filters (f) precede couplers to multimode fibers, which send light to detectors A, B, and C. The beam dump (d) collects the pump beam for safety.



Demonstrate that a superposition is formed after first beam splitter

$$BS \cdot x = \begin{pmatrix} 0.707 \\ 0.707i \end{pmatrix} \qquad \frac{1}{\sqrt{2}} \cdot (x + i \cdot y) = \begin{pmatrix} 0.707 \\ 0.707i \end{pmatrix}$$



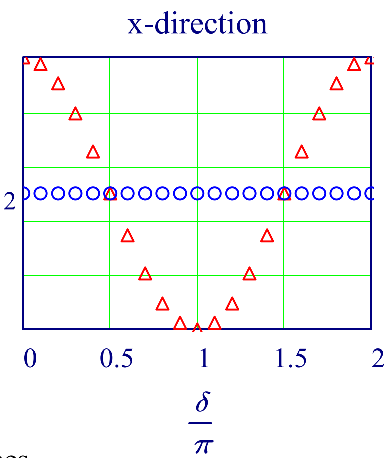
$$\delta := 0, .1 \cdot \pi .. 2 \cdot \pi$$

Confirmation that path information destroys interference.

Fig. 3. Schematic of the (a) apparatus and (b) data for the quantum eraser. The data show cases when the light leave direction not carrying path information (t along the Y direction carrying path inform

$\theta = 0$, no path information $\quad \left(\left|kronecker(X, I) \cdot MZ_{dp}(0, \delta) \cdot xv\right|\right)^2$

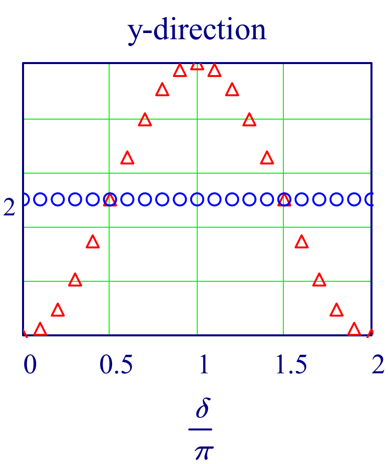
$\theta = \pi/4$, path information $\quad \left(\left|kronecker(X, I) \cdot MZ_{dp}\left(\frac{\pi}{4}, \delta\right) \cdot xv\right|\right)^2$



Kronecker is Mathcad's command for tensor multiplication of square matrices

$\theta = 0$, no path information $\quad \left(\left|kronecker(Y, I) \cdot MZ_{dp}(0, \delta) \cdot xv\right|\right)^2$

$\theta = \pi/4$, path information $\quad \left(\left|kronecker(Y, I) \cdot MZ_{dp}\left(\frac{\pi}{4}, \delta\right) \cdot xv\right|\right)^2$



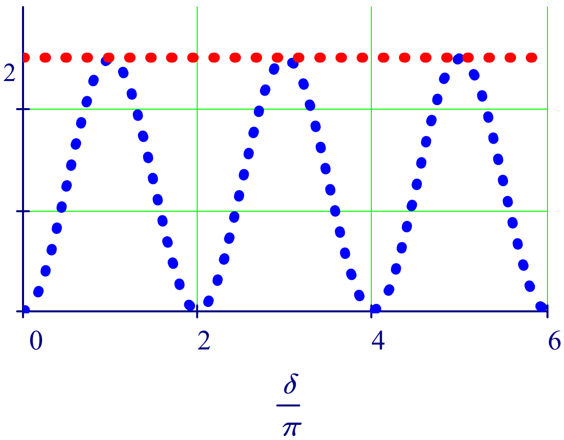
Erasure of path information restores interference. Erasers for the x- and y-directions place diagonal polarizers in those directions after the interferometer.

$$E_x := \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad E_y := \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

The x-direction has an eraser and the y-direction does not. $\delta := 0, .1 \cdot \pi .. 6\pi$

x-direction: $\left(\left|kronecker(X, I) \cdot E_x \cdot MZ_{dp}\left(\frac{\pi}{4}, \delta\right) \cdot xv\right|\right)^2$

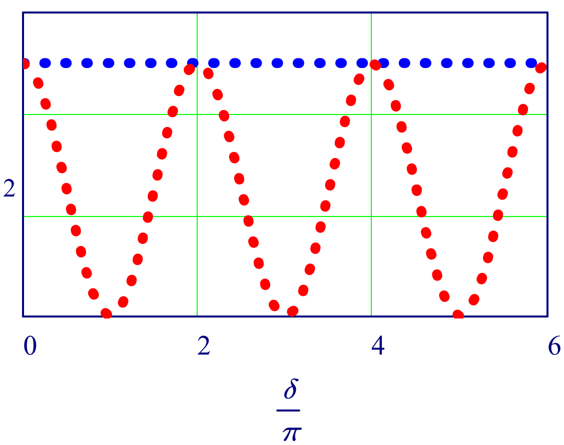
y-direction: $\left(\left|kronecker(Y, I) \cdot MZ_{dp}\left(\frac{\pi}{4}, \delta\right) \cdot xv\right|\right)^2$



The y-direction has an eraser and the x-direction does not.

x-direction: $\left(\left|kronecker(X, I) \cdot MZ_{dp}\left(\frac{\pi}{4}, \delta\right) \cdot xv\right|\right)^2$

y-direction: $\left(\left|kronecker(Y, I) \cdot E_y \cdot MZ_{dp}\left(\frac{\pi}{4}, \delta\right) \cdot xv\right|\right)^2$



For the MZ polarization interferometer diagonally polarized light enters in the x-direction, |xd>.
Tensor vector multiplication is awkward in Mathcad as is shown below.

$\left(1\right)$ $\left(0\right)$ null vector

$$\Psi_{in} := \frac{1}{\sqrt{2}} \cdot \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \qquad submatrix(kronecker(augment(x, n), augment(d, n)), 0, 3, 0, 0) = \begin{pmatrix} 0.707 \\ 0.707 \\ 0 \\ 0 \end{pmatrix}$$

No light, however, exits in the x-direction. It exits in the y-direction showing no interference effects.

$\delta := 0, .2$

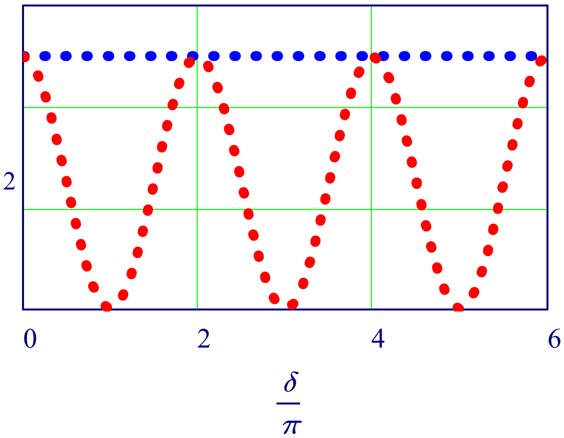
$$\left(\left|kronecker(X, I) \cdot MZ_P(\delta) \cdot \Psi_{in}\right|\right)^2 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad \left(\left|kronecker(Y, I) \cdot MZ_P(\delta) \cdot \Psi_{in}\right|\right)^2 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

Placement of a **D** polarizer in the y-direction output erases distinguishing information and interference appears.

$\delta := 0, .1\cdot\pi .. 6\pi$

x-direction: $\left(\left|kronecker(X, I)\cdot MZ_{dp}\left(\frac{\pi}{4}, \delta\right)\cdot xv\right|\right)^2$

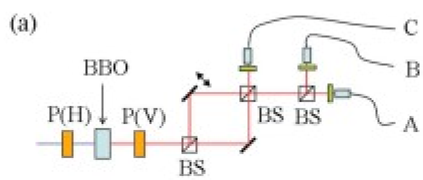
y-direction: $\left(\left|kronecker(Y, I)\cdot E_y\cdot MZ_{dp}\left(\frac{\pi}{4}, \delta\right)\cdot xv\right|\right)^2$



Calculation of exit probabilities for two photons in direction-of-propagation modes:

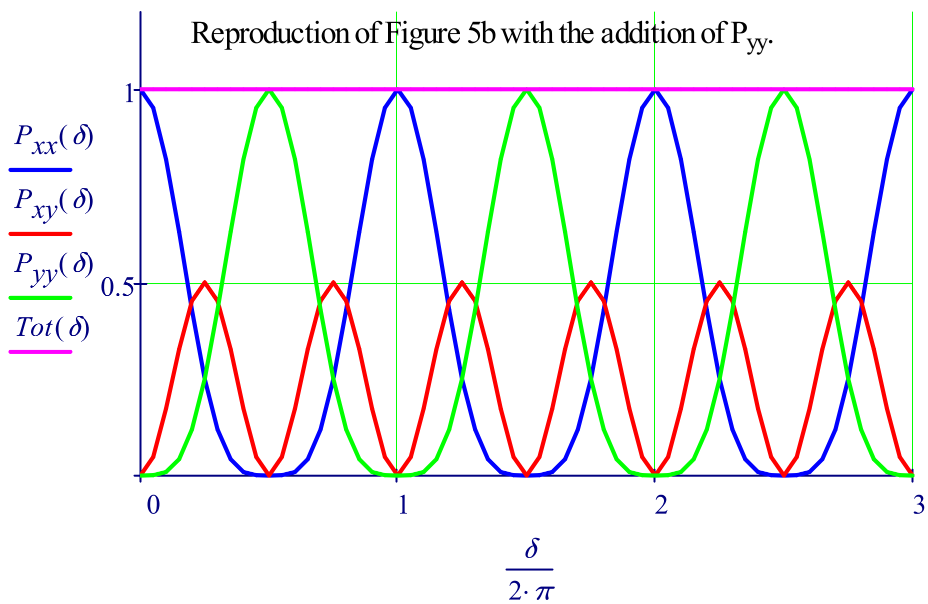
$$P_{xx}(\delta) := \left(\left|xx^T\cdot MZ_{dd}(\delta)\cdot xx\right|\right)^2 \qquad P_{xy}(\delta) := \left[\left|\frac{1}{\sqrt{2}}\cdot(xy+yx)^T\cdot MZ_{dd}(\delta)\cdot xx\right|\right]^2$$

$$P_{yy}(\delta) := \left(\left|yy^T\cdot MZ_{dd}(\delta)\cdot xx\right|\right)^2 \qquad Tot(\delta) := P_{xx}(\delta) + P_{xy}(\delta) + P_{yy}(\delta)$$



$\delta := 0, .05\cdot\pi .. 6\cdot\pi$

Reproduction of Figure 5b with the addition of P_{yy}.



"The striking result is that the (P_{xy}) interference pattern has twice the frequency of the single-photon interference pattern. Nonclassical interference shows new quantum aspects: two photons acting as a single quantum object (a biphoton)."

Hong-Ou-Mandel interference (right column, page 516):

$$BSBS\cdot\frac{1}{\sqrt{2}}\cdot(xy+yx) = \begin{pmatrix}0.707i\\0\\0\\0.707i\end{pmatrix} \qquad \frac{i}{\sqrt{2}}\cdot(xx+yy) = \begin{pmatrix}0.707i\\0\\0\\0.707i\end{pmatrix}$$

Section III.D deals with distinguishing between pure and mixed states experimentally. The pure state and it density matrix are given below.

$$\Psi_{pure} := \frac{1}{\sqrt{2}}\cdot(hh+vv) \qquad \Psi_{pure} = \begin{pmatrix}0.707\\0\\0\\0.707\end{pmatrix} \qquad \Psi_{pure}\cdot\Psi_{pure}^{T} = \begin{pmatrix}0.5&0&0&0.5\\0&0&0&0\\0&0&0&0\\0.5&0&0&0.5\end{pmatrix}$$

The density matrix for the mixed state is calculated as follows.

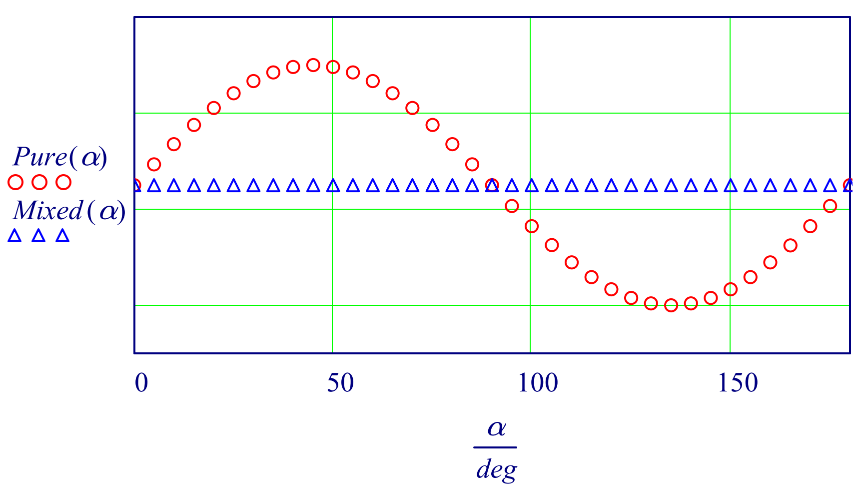
$$\frac{1}{2}\cdot hh\cdot hh^{T} + \frac{1}{2}\cdot vv\cdot vv^{T} = \begin{pmatrix}0.5&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0.5\end{pmatrix}$$

The following calculations and their graphical representation are in complete agreement with section III.D

$$Pure(\alpha) := tr\left[\frac{1}{2}\cdot\begin{pmatrix}1&0&0&1\\0&0&0&0\\0&0&0&0\\1&0&0&1\end{pmatrix}\cdot\frac{1}{2}\cdot\left[\begin{pmatrix}cos(\alpha)\\cos(\alpha)\\sin(\alpha)\\sin(\alpha)\end{pmatrix}\cdot\begin{pmatrix}cos(\alpha)\\cos(\alpha)\\sin(\alpha)\\sin(\alpha)\end{pmatrix}^{T}\right]\right] \text{ simplify } \rightarrow \frac{sin(2\cdot\alpha)}{4}+\frac{1}{4}$$

$$Mixed(\alpha) := tr\left[\frac{1}{2}\cdot\begin{pmatrix}1&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&1\end{pmatrix}\cdot\frac{1}{2}\cdot\left[\begin{pmatrix}cos(\alpha)\\cos(\alpha)\\sin(\alpha)\\sin(\alpha)\end{pmatrix}\cdot\begin{pmatrix}cos(\alpha)\\cos(\alpha)\\sin(\alpha)\\sin(\alpha)\end{pmatrix}^{T}\right]\right] \text{ simplify } \rightarrow \frac{1}{4}$$

Reproduce Figure 6 results. $\alpha := 0\cdot deg, 5\cdot deg .. 180\cdot deg$



The following calculation are in agreement with the math in the final paragraph of section IV.D.

$$kronecker\left(W_2(0), I\right)\cdot \Psi_{pure} = \begin{pmatrix} 0.707 \\ 0 \\ 0 \\ -0.707 \end{pmatrix} \qquad \begin{pmatrix} 0.707 \\ 0 \\ 0 \\ -0.707 \end{pmatrix}\cdot\begin{pmatrix} 0.707 \\ 0 \\ 0 \\ -0.707 \end{pmatrix}^T = \begin{pmatrix} 0.5 & 0 & 0 & -0.5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -0.5 & 0 & 0 & 0.5 \end{pmatrix}$$

$$kronecker\left(W_2(0), I\right)\cdot \Psi_{pure}\cdot \Psi_{pure}^{\;T}\cdot kronecker\left(W_2(0), I\right)^T = \begin{pmatrix} 0.5 & 0 & 0 & -0.5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -0.5 & 0 & 0 & 0.5 \end{pmatrix}$$

$$Pure(\alpha) := tr\left[\frac{1}{2}\cdot\begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 \end{pmatrix}\cdot\frac{1}{2}\cdot\left[\begin{pmatrix} cos(\alpha) \\ cos(\alpha) \\ sin(\alpha) \\ sin(\alpha) \end{pmatrix}\cdot\begin{pmatrix} cos(\alpha) \\ cos(\alpha) \\ sin(\alpha) \\ sin(\alpha) \end{pmatrix}^T\right]\right] \; simplify \rightarrow \frac{1}{4} - \frac{sin(2\cdot\alpha)}{4}$$

The Galvez paper shows this as [1 - sin(α)]/4, which is a typographical error. The correct answer is [1 - sin(2*α)]/4