III. Six Key Postulates of Quantum Mechanics

Quantum mechanics can be formulated in terms of six postulates. Postulates cannot be proven, but they can be tested. The five postulates discussed in this chapter provide a framework for summarizing the basic concepts of QM.

POSTULATE 1

The state of a quantum mechanical particle is **completely specified by a wave function.**The state of a physical system is represented by a normalized ket in a Hilbert space ***H***. To simplify the notation, only one spatial coordinate is considered. The Probability, ***P***, that the particle will be found at time t_0 in a spatial interval of width centered at x_0 is given by

$$P(x_0, t_0) = \Psi^*(x_0, t_0)\Psi(x_0, t_0)\, dx = |\Psi(x_0, t_0)|^2\, dx$$

The wave function must be a single-valued function of the spatial coordinates. If this were not the case, a particle would have more than one probability of being found in the same interval.

POSTULATE 2

For every measurable property of a system such as position, momentum, and energy, **there exists a corresponding operator in quantum mechanics**. An experiment in the laboratory to measure a value for such an observable is simulated in the theory by operating on the wave function of the system with the corresponding operator. All quantum mechanical operators belong to a mathematical class called Hermitian operators that have real eigenvalues. For a Hermitian operator $\hat{A}$, $\int \psi^*(x)[\hat{A}\psi(x)]\, dx = \int \psi(x)\,[\hat{A}\ \psi\ (x)]^* dx$

POSTULATE 3 (Born's Rule)

In any single measurement of the observable that corresponds to the operator, the **only** values that will ever be measured are the **eigenvalues** of that operator $\hat{A}$

POSTULATE 4

If the system is in a state described by the wave function, and the value of the observable a is measured once on each of many identically prepared systems, **the average value** (also called the **expectation value**) of all of these measurements is given by the normalized wavefunction

$$\langle a \rangle = \int \Psi^*(x,t)\hat{A}\Psi(x,t)dx$$

As we know, two cases apply with regard to *Ψ(x,t)*: it either is or is not an eigenfunction of the operator ^A. These two cases need to be examined separately. **The state space of a composite physical system** is the **tensor product** (**See Section VI**) of the state spaces of the component physical systems. If we have systems number 1 to n, prepared in state $|\psi_i\rangle$, then the joint state is of the total system is: $|\psi_1\rangle \otimes |\psi_2\rangle \otimes \cdots \otimes |\psi_n\rangle$

POSTULATE 5

The evolution in time of a quantum mechanical system is governed by the time-dependent Schrödinger equation:

$$\hat{H}\ \Psi(x,t) = i\hbar\frac{\partial\Psi(x,t)}{\partial t}$$

POSTULATE 6 (See **Section V**)

Quantum superposition is a fundamental principle of quantum mechanics. In classical mechanics, things like position or momentum are always well-defined. We may not know what they are at any given time, but that is an issue of our understanding and not the physical system. In quantum mechanics, a particle can be **in a superposition of different states**. It can be in **two places at once** (see double-slit experiment). A measurement always finds it in one state, but before and after the measurement, it interacts in ways that **can only be explained by having a superposition of different states.** A simple demonstration of superposition can be made using a beam of light that passes through a polarizing filter.

HEISENBERG'S UNCERTAINTY PRINCIPLE

It is impossible to measure or calculate exactly both the position and the momentum of an obj**ect - *Δx *Δp > h/2***

Demonstration of Heisenberg Uncertainty Principle Using Fourier Series

Domain Definition

$L := 2\pi \qquad N := 100 \qquad n := 0..N-1 \qquad x_n := -L + n\cdot\left(\frac{2\cdot L}{N-1}\right) \qquad dx := x_1 - x_0$

// Define a narrow Gaussian function

$\sigma := 0.3 \qquad f_{x_n} := \exp\left[-\left[\frac{(x_n)^2}{(2\cdot\sigma^2)}\right]\right] \qquad Norm := \sqrt{\sum_n \left[\left[(f_x)_n\right]^2\cdot dx\right]} = 0.729$

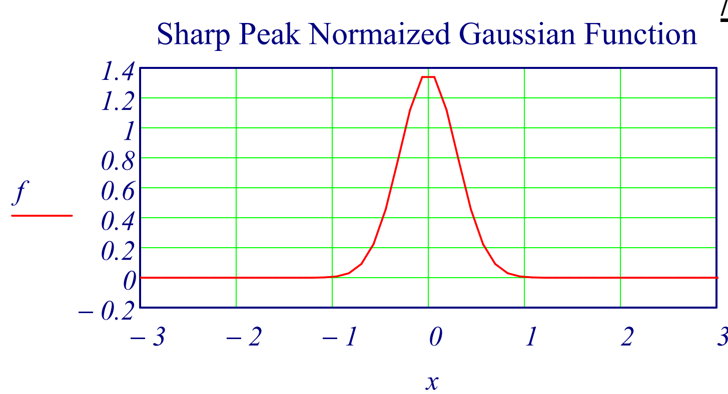
Normalized Gaussian

$f_n := \frac{f_{x_n}}{Norm}$

From Quantum Mechanics

The uncertainty in the product of the Momentum, Δp, and the position, Δx, requires that

$\Delta x * \Delta p \geq 1/2\,\hbar$



// Frequency axis

$k_n := \frac{n}{N\cdot dx}$

$\min(k) = 0$

$\max(k) = 7.799$

$k_{N-1} = 7.799$

$$Fx(fx) := \left| \begin{array}{l} f(x,y) \leftarrow 0 \\ F \leftarrow matrix(N,1,f) \\ \text{for } k \in 0..N-1 \\ \quad F_k \leftarrow \sum_{j=0}^{N-1} \left(fx_j\cdot\exp\left(\frac{-2\pi\cdot i\cdot j\cdot k}{N}\right)\right) \\ F \end{array}\right.$$

$$abs(M) := \left| \begin{array}{l} f(i,j) \leftarrow 0 \\ A \leftarrow matrix(rows(M), cols(M), f) \\ \text{for } i \in 0..rows(M)-1 \\ \quad \text{for } j \in 0..cols(M)-1 \\ \quad\quad A_{i,j} \leftarrow |M_{i,j}| \\ A \end{array}\right.$$

$F := Fx(f_x) \qquad Fmagx := abs(F) \qquad Fmag := \frac{Fmagx}{\sqrt{\sum Fmagx^2\cdot\frac{1}{N}}}$

Compute Uncertainties

The Uncertainty Principal States:

The Calculated Heisenberg Product

$x\_mean := \sum_n \left[x_n\cdot(f_n)^2\right]\cdot dx \qquad \Delta x := \sqrt{\left[\sum_n \left[(x_n - x\_mean)^2\cdot(f_n)^2\right]\right]\cdot dx} \qquad \Delta x = 0.212$

$k\_mean := \sum_n \left[k_n\cdot(Fmag_n)^2\right]\cdot\frac{1}{N} \qquad \Delta k := \sqrt{\left[\sum_n \left[(k - k\_mean)^2\cdot Fmag^2\right]\right]\cdot\frac{1}{N}} \qquad \Delta k = 3.633$

The Uncertainty Principal States:

$\Delta x \cdot \Delta k \geq \frac{1}{2}$

Emulation Verifies Heisenberg Uncertainty Principle

$\Delta x\cdot\Delta k = 0.771$